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A modeling strategy for large-scale optimization based on analysis and visualization principles

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THESIS

A MODELING STRATEGY FOR LARGE-SCALE
OPTIMIZATION BASED ON
ANALYSIS AND VISUALIZATION PRINCIPLES

by

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and
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Based on Analysis and Visualization Principles

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ABSTRACT

A modeling strategy for the validation and analysis of large-scale optimization models is defined and demonstrated. The strategy is based on nine principles of analysis and eight principles of visualization that are applied in a user controlled hierarchical structure which is customized to a particular optimization problem. For each model a set of analytic tools, such as spreadsheets and graphs, is structured to validate and verify data and analyze the model and its results. These tools can be quickly recreated with data from subsequent runs of the model and sensitivity analysis conducted and comparisons made. As a demonstration the strategy is applied to PHOENIX, a large-scale U.S. Army helicopter force planning model. The strategy incorporates available technology using commercially prepared software and a computer workstation. The application of techniques such as hypertext, data access and backward compatibility enhance the ease of use and effectiveness of this approach.

TRADEMARK NOTICES

Each registered trademark mentioned in this thesis is listed below, followed by the name of the owner of the trademark: *Improv, 1-2-3,* Lotus Development Corporation; *NeXT,* NeXT, Inc.; *VINO, LINDO, Super Macvino,* Lindo Systems; *What's Best*, General Optimization, Inc.

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I. INTRODUCTION

Mathematical programming and optimization are playing an ever-increasing role in industry and government. It is believed that this trend will continue well into the 21st century. Optimization is now being used in medicine and health care, environmental management, and securities and finance in addition to the more traditional operations research fields of transportation, production and energy. Users realize that even small improvements in operations can sometimes save millions of dollars. The widespread availability of desktop optimizers and powerful computers has already and will continue to contribute to this trend. (Hirshfeld, 1990 and Schultz and Pulleyblank, 1991)

Operations research has made significant contributions in these fields by representing a problem as a system of mathematical equations and solving for a single optimal solution. But, as advances in scientific computation allow for increasing complexity of mathematical models, it becomes apparent that they are more difficult to analyze. Many of the models now represent a recurring situation or involve a multi-period planning horizon. Such models are costly and time-consuming to develop, but this is offset by

their usefulness as planning tools which can be used over and over again. Long-term model use frequently results in second-generation users, who are not familiar with the original model development, being required to execute and analyze follow-on runs. Even model developers may have difficulty retaining the intricacies of the problem after significant time lapses between runs.

Historically, models of multi-faceted problems had to have been cut down to size and drastically simplified to bring them within the scope of human thinking powers, and computing technology. This is no longer a satisfactory method of coping with many of the complex dynamic systems being modeled and solved today. Due to the rapid advances in the storage and processing capacity of today's computers and the increased prevalence of human understanding and use of these mechanical abilities, it is no longer necessary to simplify these models. Today's models are extremely elaborate, modeling a multitude of intricate situations through the use of thousands of linear and non-linear constraints and real, and integer variables. Larger and more complex problems can now be modeled and solved faster and more efficiently as computers have the capability to handle larger data bases, more constraints, more complex computations, greater storage, and more detailed displays.

Just as the models are no longer simple, the decisions associated with the problems are now more diverse. Specifying an objective function value is often an attempt to minimize or maximize some ambiguous measure of effectiveness. Similarly, many of the bounds on constraints are subjective and not easily defined as they are often measures of some intangible quality such as efficiency, durability, or some other performance criteria. The use of elastic constraints compensates somewhat for these subjectively assigned constraint bounds. It allows the constraints to be violated but only at some cost internal to the model.

Due to the complexity of problems being solved through optimization an analyst needs to pursue and explore formulations and solutions which most accurately describe the situation and provide not only feasible, but practical courses of action. Technological advances in the past decade have resulted in a many-fold increase in speed and availability of computer computational power and significant decreases in the cost to solve large models. This has supported the analyst's ability to process data for several optimal solutions or a range of feasible solutions, enhancing the traditional approach of pursuing a single optimal answer to a question or problem. Comparisons of these multiple optimal solutions can be made and the effects of changing constraint

conditions can be fully explored. As a necessity, analysts have become far more capable in their abilities to conduct sensitivity analysis of the effect of data changes on the solution even when thousands of constraints are involved.

Technology, however, has not yet reached all aspects of the modeling process. Large, complex models require voluminous input and generate voluminous output. Multiple runs of the model compound this situation. The analysis required to obtain the best possible decision on the most realistic model possible must continue to become more sophisticated. Today's analysts are better able to take realistic account of the incomplete information and inconsistency of a complex and changing world and of the compromises and approximations that must be made to fit real-world problems into the quantitative terms of a model. As always, analysis is the key to both the model and the solution of large-scale optimization. (Jones, 1988, p. 891; Linstone, 1985, pp. 77-84; and Simon et al 1987, pp. 11-15)

The challenge now is to combine these aspects of modeling into a total optimization system which will meet the need for "more complete and integrated optimization solutions comprised of model preparation facilities, analysis tools including visualization, and easy access to enterprise-wide data in addition to the solver

capability." (Schultz and Pulleyblank, 1991, p. 21). As the problems become more complex and increasing technology allows faster solutions, the limiting factor in optimization is the time and energy required of the analyst to sort through input and output data to find and interpret the solutions.

A naive view considers optimization to be a forward progression through four basic stages: problem definition, data collection, model formulation and analysis of the results. Analysts realize that there is much more to effectively employing large-scale optimization models to solve today's complex problems. There is a widely used, though seldom recognized, interactive optimization process in which analysis is conducted at each of these four steps. As data is collected, it must be verified and validated. The model must be compared with the data to ensure feasibility and both the model and its solution must be compared against the problem to ensure fidelity. Sensitivity analysis is accomplished by comparing the results of multiple runs of the model.

The modeling strategy for interactive optimization presented in this thesis seeks to harness available technology to vastly improve and speed this analysis. It is an approach to the interactive optimization process that consists of designing a series of analytic tools such as graphs and spreadsheets customized to a particular

application. These tools can then be used by an analyst on subsequent runs and easily extended as necessary to new information and considerations. It is especially helpful in the analysis of multi-year and recurrent models and it reduces the information gap and learning curve between periodic uses and for second-generation and subsequent analysts. The strategy can be applied to a large variety of models. It is based on a set of analysis and visualization principles that are uniquely tailored to a particular model and driven by that model and the needs of the analyst. The implementation of this modeling strategy will reduce the time and effort to validate data and interpret solutions allowing the analyst to better pursue model development and analysis to make better decisions.

These analysis and visualization principles are presented in Chapter II along with a more detailed discussion of the interactive optimization process. Background on human visual processing and computer technology is discussed in Chapter III. Chapter IV is a description of the user for which this strategy is developed. Chapter V outlines the PHOENIX model, a U.S. Army helicopter force planning tool. Chapter VI demonstrates the application of the modeling strategy to PHOENIX. Chapter VII describes the future of the PHOENIX model. The conclusions are presented in Chapter VIII.

II. THE MODELING STRATEGY

A. INTERACTIVE OPTIMIZATION PROCESS

In order to develop an optimization modeling strategy an in-depth understanding of the interactive optimization process is required. As conceptualized in Figure 1, the four basic parts of the process are problem definition, data collection, model formulation and analysis of results. The main relationships among these parts are depicted by the forward arrows. The backward arrows represent the relationships that are more subtle, yet crucial, in the optimization process.

As data is collected it must be consolidated, verified and validated. Visual inspection of the raw data files or printouts is simply not feasible. The input data often originates at several sources and needs not only to be verified for accuracy, but also to be compared for consistency among sources. Without correct input data, the results of the model are worthless. The verification and validation of input data needs to be an integral part of optimization. It must be done both as the data stands alone and as it is linked together with the model.

All aspects of the problem that are critical to the solution must be enumerated and accounted for in mathematical formulas. Modeling requires that an objective function and constraints, often qualitative terms, be quantified in some way. This presents a difficulty in that real world situations are complex dynamic systems which can seldom be translated into black and white values and it is often impossible to establish a hierarchy of these values. If there are mutually exclusive requirements, elastic constraints are introduced to ensure that the unfulfilled requirement is at some cost internal to the model.

Both the model and its solution must be checked against the actual problem to ensure fidelity, validity and practicality. If not adequately representative of the situation being modeled, changes may need to be made in the data collection, model formulation, or even problem definition. The analyst must often explore alternate solutions obtained by varying the input parameters and constraint conditions to reflect the dynamics of a complex problem. Even small changes made to enhance the application of the model can indicate an entirely new course of action.

In order to conduct sensitivity analysis of large-scale problems, the results of multiple runs must be compared. The voluminous data produced by each run requires the same in-depth

analysis as the original output. Thorough comparisons between successive runs highlight the advantages of one over another and aid in the selection of one optimal or a range of feasible solutions.

Faster more efficient computers allow the solution of these large-scale problems. Their final results can be displayed and presented very effectively using a variety of sophisticated presentation tools. Technology, however, has not yet reached the relationships in the interactive optimization process represented by the back arrows. There will not be a general way to bring technology to these relationships since each model is an individual problem with a hierarchy of analysis consisting of unique questions and decisions to be made about the optimization output. The modeling strategy developed in this thesis can help to bring technology to the interactive optimization process by defining a set of analysis and visualization principles that can be applied in different combinations to all models.

B. PRINCIPLES OF THE MODELING STRATEGY

The analysis and visualization principles that are the basis of the modeling strategy will be briefly explained in this section. Although it would be possible to more fully develop each of these principles, a more effective way to present the strategy is to apply it to a real, contemporary, complex, and important model. The

strategy will be applied to the PHOENIX model in Chapter V. Each principle will be demonstrated, however the focus is not on individual principles, but on the combinations, interactions and compromises among these principles as they are applied to an actual model.

1. Analysis Principles

- (A1) HIERARCHICAL STRUCTURE. Analysis progresses from a broad overview of the objective function value down to a single piece of information. The specific hierarchy will be designed by the analyst and will depend on the nature of the problem. Levels may be skipped and crossovers between branches can occur to meet the needs of the analyst.

- (A2) USER CONTROL. The analyst is able to navigate the hierarchical structure to answer the questions that arise as part of a specific analysis.

- (A3) DATA ACCESS. At all points in the analysis process, the analyst must have easy access to specific data values and the relationships among those values.

- (A4) MULTIPLE REPRESENTATIONS. Different aspects of the same data are extremely valuable in revealing information about the relationships among data. These varied representations are easily constructed and readily available.

- (A5) SIDE-BY-SIDE COMPARISONS. Side-by-side comparisons of similar information enhance the analysis process by decreasing the amount of decoding that is required and by highlighting differences.

- (A6) BASE CASE. In model development and sensitivity analysis, a base case that is believed to best represent the problem can be used. Results of other runs are compared against the base case.

- (A7) CUSTOM MODEL. The nature of the problem and the questions inherent to it drive the set of representations that are used for analysis of that model.

- (A8) EXTENSIBLE. Changes made to a particular representation or piece of information filter through all representations that contain that information.

- (A9) BACKWARD COMPATIBLE. Any additional representations or changes to existing representations made during analysis of a run are automatically added to any previous runs of the model.

2. Visualization Principles

- (V1) REPRESENTATION DRIVEN GRAPHS. The results of time-series models are most often displayed with the time periods on the horizontal or x-axis. This fits in with the concept of time moving forward and the general perception of the forward direction as the one from left to right. Alternately, a model which seeks to find the best combination of ingredients in order to make some new product may be best represented by filled bar charts where the height of the bar represents the new product. Pie charts could also be used but are generally not considered good analysis tools due to the difficulties encountered in perceiving differences in angles and slopes (Tufte, 1983, p. 178 and Cleveland and McGill, 1985, p. 829). Stacked bar charts are also very effective in showing different combinations of things that make a whole.

- (V2) ZOOMING. The ability to focus or zoom in on areas of interest or out to the big picture is crucial in the analysis process. The focus could be on certain time periods, on information pertaining to a particular constraint, or even on the objective function value. It allows the analyst to pursue a separate train of thought or to redirect analysis in a certain direction.

- (V3) DATA ACCESS. The analyst must also have simple, easy access to the data behind the graphs. Whereas the graph provides a way to view the information in relation to other information in the model, the actual data is necessary for quantitative analysis.

- (V4) APPROPRIATE DISPLAYS. Not all information is most effectively displayed in graphic form. The analyst must decide when a graph is needed and when displaying the data in tabular form is more informative.

- (V5) SIMPLICITY. Visualization tools must be designed to reveal information rather than draw attention to the display. They should avoid distortion of the data and allow the analyst to interpret it. (Tufte, 1983, p.14)

- (V6) CONSISTENCY. Graphs that are consistent in size, coloring and labeling with other graphs which display similar information reduce the amount of time required for interpretation.

- (V7) MULTIPLES. An especially effective method to display and highlight both subtle and extreme differences is the use of small multiples which are a series of graphics, each showing the same combination of variables, indexed by changes in another variable. Since the design stays constant, the analyst's attention can be devoted entirely to the changes in the data (Tufte, 1983, p. 170).

III. BACKGROUND

A. HUMAN VISUAL INFORMATION PROCESSING

Extensive research has been conducted on human visual perception and information processing. The physiology has been explored (Marr, 1982, pp. 4-15) and numerous experiments conducted (Powers et al, 1983 and DeSanctis, 1984) to determine the benefits of using representation (graphics and tables) versus description (text) to enhance the human's ability to process, recall, and analyze visual information. Under a variety of circumstances and experimental conditions, some studies conclude the superiority of graphical presentation over text or tables and some determine that no one method can conclusively be determined superior. When evaluated, Powers et al determined that the combination of graphical and tabular data was more effective than either method used alone (1983, p. 558). Dependent variables evaluated in these studies differed, but a typical list comes from DeSanctis (1984, p. 468):

- Interpretation accuracy;
- Problem comprehension;
- Task performance;
- Decision quality;

- Speed of comprehension;
- Decision speed;
- Memory for information (recognition and recall);
- Viewer preference.

Further research more specifically explores effective techniques for presenting quantitative information. In his books, Edward Tufte asserts that "well-designed data graphics are usually the simplest and ... most powerful of all methods for analyzing and communicating statistical information" and that the general principles of effective design are universal, not tied to language or culture (1983, pp. 8-9 and 1990, p. 10). "Often the most effective way to describe, explore, and summarize a set of numbers - even a very large set - is to look at pictures of those numbers." (Tufte, 1983, p. 9) To communicate complex ideas with clarity, precision, and efficiency, Tufte lists the important elements of graphical displays (1983, p. 13):

- Show the data;
- Induce the viewer to think about the substance rather than about methodology, graphic design, the technology of graphic production, or something else;
- Avoid distorting what the data have to say;
- Present many numbers in a small space;
- Make large data sets coherent;

- Encourage the eye to compare different pieces of data;
- Reveal the data at several levels of detail, from a broad overview to the fine structure;
- Serve a reasonably clear purpose: description, exploration, tabulation, or decoration;
- Be closely integrated with the statistical and verbal descriptions of a data set.

Cleveland and McGill (1985, p. 828) caution against graphs that are too elaborate or technologically sophisticated. They have studied the link between graphs and the human visual system and conclude:

> When a graph is constructed, quantitative and categorical information is encoded, chiefly through position, shape, size, symbols, and color. When a person looks at a graph, the information is visually decoded by the person's visual system. A graphical method is successful only if the decoding is effective.

Norbert Enrick (1972, p. 2) cites the values of "well prepared charts and graphs" as creating interest, clearly portraying relationships, saving time, saving space, providing a synoptic overview, unearthing hidden factors, and enhancing thought processes, analysis, ideation and creativity.

In the field of operations research, the primary applications of graphic techniques to date have been in the areas of simulation and data analysis. While both are applicable to optimization, surprisingly little has been done to incorporate graphical methods into the analysis process for large-scale optimization models. But

as more complex and increasingly larger optimization models and more sophisticated operations research techniques are developed "understanding the behavior of the underlying system, detecting trends, debugging and validating the model become more challenging." (Jones, 1988, p.6) Graphical and visual techniques and user interaction in the modeling process can be employed to assist significantly in optimization analysis. Incorporating these techniques more prominently into operations research has been cited as a need in the current growth of the field of operations research, and their potential benefits are inestimable. (Jones, 1988, p.7; Bell, 1985, p.31; Hurrion, 1986, p.286)

B. REPRESENTATION/VISUALIZATION

With the increasing volume of input data and results that can now be produced by large-scale optimization, the problem analysts face has changed from making limited calculations to being able to understand and interpret the masses of data produced. Effective representation is a key factor in the abilities of the analyst to validate and analyze this information. Text and tables have long been used by optimizers for conveying the data and results of their algorithms. But perhaps the most effective and universally understood means of representation is through the use of graphics. Computer graphics programs that are used to help scientists

visualize and thereby better understand their research problems are categorized as scientific visualization (Rivenbark, 1989). Along the lines of the old adage "a picture is worth a thousand words", these visual images can convey more information and reveal aspects and relationships not as easily discerned by analyzing formulas or perusing numerical values. "Using computer generated images and human vision in scientific visualization ... (can) convey a tremendous amount of information in a short period of time." (Nielson, 1989, p. 10). The strength of visualization is its use of the greatest processor available: the human brain, according to Lloyd Treinish, a computer scientist with NASA's National Space Science Data Center (NSSDC), because it "takes advantage of the inherent power of the human visual system." (Rivenbark, 1989, p. 36)

Craig Mundi, Research and Development vice president for Alliant Computer Corporation, Littleton, Massachusetts, divides visualization into two types: statistically produced data sheets from which the user, or computer, extracts a static graphical representation, and models that the user dynamically manipulates in real time (Jones, 1988). This real-time animation is becoming more frequently utilized in the field of operations research for animation of algorithms and simulations models. Animation allows

the analyst to interact with and thereby redirect or change the actual problem during its solution. The solution or the algorithm can also be animated showing the user the actual process behind the model and the steps involved in solving it. Currently, however, technology cannot accommodate the animation of large-scale optimizations with their multitude of variables and constraints. Three-dimensional computing has improved Operations Research visualizations greatly, but scientific computing has yet to master graphing the thousand-dimensional and the human mind would have difficulty comprehending it.

Optimization does not now allow run-time interaction to redirect the model or make changes during computation. The reason for this is found in the Simplex procedure involved in linear programming. The assortment of variables and constraints that make up the optimization algorithm form, in effect, a multi-dimensional polytope whose extreme points bound the feasible solutions. The Simplex optimization process searches these points for the optimal solution. Whether using the traditional Simplex algorithm which progresses from one adjacent vertex to another until there is no improved point, or the more recently developed Karmarkar algorithm (Hamilton and Stein, 1989, p.36) which employs a shortcut to work through the center of the polytope instead of on the surface, any

run-time interaction would change the composition of the polytope and invalidate the systematic optimization process. The use of static graphical representations of visualization are more appropriate, then, to improving the portrayal of large-scale optimizations.

What visualization can provide to large-scale optimization is enhancement and clarity of voluminous input and results. It can be used as a vehicle in focusing and directing analysis. The strategy described here employs these static visualizations in an interactive environment where analysis and changes can be incorporated between successive runs of the optimizer to explore alternate decision strategies. The development of graphics programmed directly into the output production excuses the analyst from having to become a computer programmer and from performing labor intensive data manipulations. It speeds the preparation of graphic representations of the model and eliminates the need to "reinvent the wheel" as the analyst elects to display different formats, change input values and parameters, and validate and make comparisons on multiple runs of the optimization (Rivenbark, 1989).

These changes serve to produce better problem solving in the field of operations research and enhance the credibility of analysts in their work with managers and decision makers who prefer a more

socio-technical approach to problem solving. To fully realize the potential of mathematical optimization, the effectiveness of algorithms and their implementation must be developed to solve the actual problems of the user (Schultz and Pulleyblank, 1991). By capitalizing on the speed and power of delivery systems in the development of a total optimization system, the modeling strategy will greatly enhance the capabilities and latitude of the analyst.

C. COMPUTER TECHNOLOGY

The great strides made in computer technology have resulted in a decline in price coupled with a rise in capability. This puts extremely powerful machines within easy reach of analysts and researchers. These machines can solve exceptionally large, complex problems in relatively short periods of time as they are adequately equipped with memory to store the tremendous amount of data and the intricate relationships among its elements that these problems demand. Along with this increased power come fast enhanced graphics that can provide the user with high quality visualization products for both analysis and presentation. This is especially important for operations research, and even more so for optimization, as it allows the modeling, solving and analysis of production sized problems with a degree of fidelity that has never before been possible. Gregory M. Nielson, in an article in *Computer* magazine

(1989, p.10), captures the advances of computer technology and highlights the direction for the future:

> Advances in scientific computation allow increasing complexity of mathematical models and simulations. This results in a closer approximation to reality, which enhances the possibility of acquiring new knowledge and understanding ... The problem is to convey all of this information to the scientist to effectively use human creativity and analytic capabilities.

Of the newly affordable computer systems, the workstation may have the most profound effect on the operations research analyst. In addition to having the power and speed of the latest technology, it has several other characteristics which enhance the capabilities of an analyst. One advantage is the high speed architecture which allows quick transfers among many different environments such as the model, the spreadsheet and the graphics. Additionally, most workstations employ a windowing system which provides a set of programming tools and commands for building the menus, windows, and dialogue boxes that appear on a screen. These two features alleviate the difficulties and inconveniences of switching computer environments and facilitate the simultaneous use of multiple applications for comparison and consolidation.

While workstations are self-contained computers, they are also an integral part of the networking concept. In a network, information is shared among all the systems which are linked to each other via this network. In addition to sharing information,

networks can also share the work required to solve large-scale problems. One computer is in primary control of a problem but can distribute solving tasks to other computers, including other workstations, micro-computers and even a mainframe. The distribution is based on both the capability and availability of the machines on the network. When a task is completed, its solution or status report is sent back to the controlling computer for consolidation with the rest of the problem until the entire job is complete. This process maximizes the effective use of the computers and minimizes the time required to solve a problem. The interactive optimization process discussed in this thesis does not directly rely on networked problem solution, but there is potential for links with other systems to take full advantage of a network during the course of an optimization.

IV. DESCRIPTION OF TARGET USER

Currently, off-the-shelf software exists to bring the methods of optimization to the general user. Several packages, such as *VINO* and *Lotus 1-2-3* utilize spreadsheets and, with simple instructions for input of objective function and constraint values, will produce an optimal solution without the requirement of user comprehension of optimization principles. Perhaps one of the most capable products for simple linear optimization is *What's Best* which integrates the flexibility of the *Lotus 1-2-3* spreadsheet with the power of *LINDO* (Bodily, 1986, pp. 41-42). These programs, while useful for more simple tasks, are not sophisticated enough for the complex problems encountered by operations research analysts.

The process developed in this thesis will address optimization at this higher level. It is designed for a sophisticated user trained in operations research and analysis. It requires an in-depth understanding of the theory and mechanics of mathematical programming, including the principles of sensitivity analysis. A working knowledge of data analysis techniques with emphasis on graphical representations is also essential. This system is not intended for a casual user. A level of involvement is expected that

would justify the construction of a project-specific application of the process developed. It is envisioned that the analyst will have a long-standing relationship with a particular project and with optimization projects in general. This will ensure that the process is used to its fullest extent and that the user will derive the maximum benefit from it. Additionally, many long-term projects are designed for use as recurring decision support models. The techniques, therefore, must be sufficiently simple and generic to be successfully transferred among qualified users.

As a minimum it is expected that the user will have:

- 6 semester hours of graduate credit in linear programming and integer programming methods.
- 3 semester hours of graduate credit in data analysis techniques.
- familiarity with spreadsheets and their related graphics packages.
- experience with a computer workstation environment.

V. THE PHOENIX MODEL

A. PURPOSE

PHOENIX is currently used by the U.S. Army Concepts Analysis Agency (CAA) as a decision aid for helicopter force planning. It was developed in late 1987/early 1988 in response to the realization by the U.S. Army that it had no comprehensive plan for modernizing its helicopter fleet. The helicopter fleet was composed of mostly Vietnam-era aircraft which were nearing the end of their useful lives. The Army Aviation Modernization Trade-off Requirement (AAMTOR) study (Force Systems Directorate, 1988) was commissioned to develop a comprehensive force planning decision aid and the PHOENIX model was the result. As described by Brown et al (1991), PHOENIX "captured complex procurement and modernization tasks in an optimization-based decision support system ... which recognizes yearly operating, maintenance, retirement, service-life extension, and new procurement costs while enforcing constraints on fleet age, technology mix, composition and budgets over a multi-year planning horizon". The final report of the AAMTOR study describes the model development, data collection and analysis in full detail.

B. DESCRIPTION

The portions of the model which are critical to the understanding of this thesis are summarized in this section and the following one. (Force Systems Directorate, 1988). PHOENIX is a mixed integer linear program (MILP) variant of a classic operations research optimization problem, the equipment replacement model. Since the real scenario is more complex than the classic problem, the PHOENIX model is also more elaborate (p. 3-1). PHOENIX addresses such concerns as multiple missions of the Army aviation fleet, budgetary limits, fixed costs associated with production and multiple criteria for mission fulfillment that are not accounted for in the basic equipment replacement model. The PHOENIX model was solved using a commercial quality optimization package, the "X-system" (p. 3-10). The most difficult scenario that was solved was over a 25 year planning horizon. It contained 288 binary-valued decision variables, 9579 continuous decision variables and 3737 constraints (p. 3-10).

C. GOALS AND ASSUMPTIONS

The objective of the PHOENIX model is to minimize the sum of the operations and maintenance annual expenditures and the penalties associated with constraint violations. The annual budget and

mission fulfillment requirements are included in the constraints of the model. The model determines (p. 5.1):

- When (time period) aircraft production lines begin and end production, or if they do at all.

- How many aircraft are purchased from each production line in each time period.

- How many operational aircraft in a cohort are improved through a Service Life Extension Program (SLEP) in each time period.

- How many operational aircraft in a cohort are retired in each time period.

Some of the key assumptions of the model are (p.1-8):

- Aircraft are purchased and supported as cohorts composed of all aircraft of the same model produced during the same time period (year).

- Aircraft age can be managed in years (age of the airframe) vice its actual flying hours.

- Aircraft are paid for in budget year t and delivered in budget year t + L, where L is the lag time for production.

- All expenditures are planned for in constant dollars.

- Monies not committed in budget year t are not carried forward to subsequent years.

- Annual requirements for aircraft include float, training and operational needs.

- Fixed costs associated with opening, maintaining and closing an aircraft production line are significant and must be included in long-range plans.

- Only one production line may exist for a particular type and model of aircraft in any time period. Certain models are predecessors to others on an individual production line and can't be produced concurrently, while production lines

producing different aircraft may operate simultaneously in a time period.

Production lines have minimum and maximum sustaining rates of production.

The user of the model specifies different requirement, resource, and policy parameters which are included in the constraints of the model and which can influence its solution (p. 5.1). The requirement parameters are:

- The number of time periods to consider.
- The minimum and maximum number of aircraft necessary to satisfy each mission in each time period.

The policy parameters are:

- The maximum useful life (time periods) of each aircraft in a mission fleet in the model.
- The minimum fraction of each mission fleet to be composed of high-technology aircraft in each time period.
- The maximum average age of each aircraft in a mission fleet in each time period.

The resource parameters are:

- The minimum and maximum budget available to spend on aircraft procurement, aircraft operations and maintenance, and on aircraft retirement in each time period.
- The existing inventory (including year of manufacture and number of aircraft) and their technology (high or low) and cost characteristics.
- The technology and cost characteristics of new aircraft designs and design improvements.

- The production line characteristics, including fixed costs, production capacities and limits, types of aircraft which can be produced and the constraints in opening and closing dates.

D. IMPORTANCE OF MODELING STRATEGY TO ANALYSIS OF PHOENIX

PHOENIX is a real, complex, specific, important, contemporary, subjective model. Decisions that are made based on its results can influence the spending of billions of dollars. Brown et al. (1991) discuss some of the difficulties encountered during the modeling and analysis of this problem. Many are problems that would have been alleviated by use of the modeling strategy described here. In particular, they discuss the complications faced in both gathering and validating data from multiple sources and in uniting this data with the actual model formulation.

The selection of a tangible and realistic objective function was difficult in light of the many different criteria for mission success and budget limitations. They also had the difficult task of developing a penalty system that accurately reflected the relative importance of each of the measures of mission accomplishment incorporated into the model as elastic constraints. The assignment of many constraints was subjective with the analysts using "corporate wisdom ...[to] characterize the current fleet status, costs, and likely consequences of future procurement and manufacturing options" (Brown et al, 1991). Since this was a

complex problem that had never been modeled before and because its impact would be so far-reaching, there was great concern about ensuring all relevant parameters were accounted for and that the model was a true depiction of the problem. The situation was further compounded because the model developers were working on separate parts of the model on opposite sides of the country.

The scope of the PHOENIX decision is broad, influencing not only the multi-year planning of Army helicopter procurement but interacting with the modernization of other Army and Department of Defense organizations. The helicopter program is competing for a portion of a fixed budget. PHOENIX is a high visibility model that must be justified at all levels, from user to congressional, and presented to a variety of audiences from technicians and analysts to generalists in Congress.

Because of its importance and the dollars involved, it is critical that the analyst explore a large variety of alternate solutions obtained by changing the resource, policy and/or requirement parameters. It is necessary to fully analyze each of the plans since one alternative could provide a lower objective function value than another plan, but not be considered a better plan due to the constraints that were violated and the corresponding money assessed in penalties. A keen understanding of the

implications involved in each of the alternate plans is critical to the solution of the problem.

Sixteen instances of the model were run as part of the initial study and two were presented in the report. Results of each scenario had to be put into tables, then converted into graphs. The goal of the strategy presented here is to use the principles of analysis and visualization to augment the validation and analysis process. More runs could be analyzed with more conditions considered. This would yield greater confidence in the decision recommended and help to prepare the results for presentation to their various users.

VI. IMPLEMENTATION OF THE STRATEGY

A. TOOLS

The analysis and visualization principles presented in Chapter II are implemented through a series of spreadsheets and the associated graphics. This particular application is built using *IMPROV* spreadsheets and *Presentation Builder* graphics on the *NEXT* computer, a low cost ($5000) workstation. Other visualization software could be used subject to user preference and availability. The specific software packages are not important, rather the flexibility and enhanced capabilities that they render allow more time to be devoted to analysis rather than to data manipulation. This is especially important given a recurring model such as PHOENIX which acts as a decision aid in a continuously updated planning process.

The analyst can customize a series of worksheets and graphs that is generated each time the optimization is run. It is envisioned that each optimization model would dictate a set of spreadsheets and graphs that are uniquely suited to the analysis of that project. These tools become part of the optimization process and are available to be used and amended as necessary during the

analysis. This streamlines the analysis process when multiple runs of the model are conducted. The same information for each run is in the formatted set of worksheets and graphics, facilitating the analysis process and the comparisons between successive runs.

The use of spreadsheets and their associated graphics as tools in the optimization process provides significant advances in the flexibility of the analysis. The data is easily manipulated via simple formulas opening possibilities for recognizing new information or new aspects of information that can be derived from the raw input data and the results of the model. The data can be combined in many different ways or separated into its individual pieces as the analyst sees fit. In this way, data is manipulated to reveal information.

Both the worksheets and the graphic packages are three dimensional. Additionally, any changes to the data in a spreadsheet ripple through all associated spreadsheets and graphics enabling the analyst to see how the change effects the other aspects of the model. This can be used as a stepping stone in model validation and exploration in a limited "What if?" scenario. The results of the changes may indicate to the analyst what changes to make to input data or constraints.

Whereas the *IMPROV* spreadsheets and *Presentation Builder* graphics are exceptionally capable, they do not as yet incorporate all the technology that is currently available in similar software packages. Since this technology will be available, it is included in the application presented in the next section. In particular, hypertext, a software system that supports special links within a single window and between pairs of windows, is applied. Hypertext allows a user to enter a software package and program changes to it that enhance the capabilities of the software and tailor it to the user's needs. Here the term hypertext will refer to the capability for the user to zoom in and view only selected information in a window or easily move from viewing one window to viewing another, usually by mouse clicking on a special button. Additional aspects to hypertext systems are discussed in Conklin (1987).

An application of the analysis and visualization principles to a sample run of PHOENIX with a 20-year planning horizon and a 2% real budget growth rate demonstrates the modeling strategy. The thought processes and decisions an analyst might make as part of the interactive optimization process conceptualized in Figure 1 are described. The caption under each graph includes the principles that the graph most remarkably illustrates. Some of the principles,

such as a customized model and data access, apply to all the graphs and are not specifically noted in each individual graph.

B. VALIDATION AND VERIFICATION

Of initial importance to the analyst is the assurance of the validity, accuracy and consolidation of the input data. The spreadsheet environment is one of the most organized and effective means to consolidate and compare data. Data fields can be directly imported into a spreadsheet. Other programs, most importantly in this case, the optimizer, can directly read from these spreadsheets. The spreadsheets are, in turn, directly linked to graphics presentations where changes made to data in the spreadsheet are automatically communicated to the graphics.

Using the appropriate graphs, the analyst can look for trends and discrepancies from trends as well as outlier values in the data. It is generally obvious from the nature of the data what trends should be exhibited and any deviation from these trends should be examined. For example, budgets over time tend to increase or remain constant, so any short term decrease would be a signal to the analyst for further exploration. It could be indicative of an actual trend or it may simply be the result of incorrect data. The labels in the spreadsheets facilitate the verification of isolated values and sparse elements of information. This process is

ultimately more effective and less time-consuming than manual scrutiny of raw data files. The input data for the PHOENIX model consists of both isolated values and sparse elements; those that will display a trend and those that are unrelated pieces of information that need to be individually scrutinized. For example the budget limits are easily verified for consistency in Figure 2.

It is important to realize that all dollar figures in PHOENIX have been adjusted for inflation and are displayed in 1988 dollars. Thus, it is the assumption in this run of the model that there will be an increase in the budget maximum limit over the time horizon of the model where the lower limit will remain constant after 1995.

Associated with the budget are several growth rates that can affect the problem solution. They are the budget growth rate, the inflation rate and the O&M growth rate displayed in Figure 3. In addition to the actual values, it is interesting to see these magnitudes in relation to each other and to verify that this is consistent with the trends in current government spending.

Three other areas which should display trends are in the mission capability areas of maximum age, high-technology fraction and force requirements, shown in Figure 4. Since the goal of PHOENIX is to ensure that the Army helicopters are sufficiently capable of performing their mission in the future, it is logical to

assume that the trends should be for helicopters that are better equipped technologically with a lower maximum age. These accomplishments would allow for a reduction in current force size to some constant level. See Figure 4. Any deviations from these trends, such as the slight dip in high-technology fraction goals in 1999, are apparent to the analyst who will then attempt to discover their cause. Line graphs such as these vastly reduce the amount of time devoted to data validation and become extremely useful in analysis and presentation of results of the model.

When graphically displayed, time series data such as the scheduling information shown in Figure 5 becomes much more informative. Recalling that PHOENIX demands that only one production line be open at a time for each aircraft mission and that certain lines are predecessors to others, this graph is a ready reference for determining when each production line may open and close.

Some other information which requires verification is not as meaningful when depicted graphically since it shows no trends or consistencies. In PHOENIX, this data includes information on aircraft such as purchasing cost, last high-technology year and the other information shown in Figure 6. The spreadsheet format greatly enhances the ability of the analyst to both understand the

information and verify its accuracy. The analyst also has access to other information in spreadsheet format such as production line capacities and initial force compositions. Simple spreadsheet formulas can easily be used to compare and consolidate information.

After this validation and consolidation is complete and any necessary corrections made, the optimization model is ready to run. Spreadsheets and graphs are again utilized to display the results as part of the interactive process. The particular views that are used here have been selected as part of the PHOENIX optimization process but can easily be manipulated to highlight any information which is of interest.

C. SINGLE-RUN ANALYSIS

Once the optimization is run, the analyst begins the analysis of overall results. The progression through the analytical hierarchy may vary by analyst and situation and the visualization tools can be tailored to accommodate this. Of initial interest is the objective function value. Recall that in PHOENIX the objective function is to minimize the sum of the O&M costs and the penalties. Look at both the total objective function value and its components in Figure 7. Due to the multi-year planning horizon of PHOENIX, the analyst may gain more insight from examining the annual components of the objective function value in Figure 8. At this point, the

analyst could choose to proceed in one of the two directions depicted in the diagram in Figure 9.

In an attempt to determine whether the model is accurate and provides the "best" solution, the analyst must assess whether penalty values are realistic as assigned and if penalties taken within the model are appropriate in finding an optimal solution. If the penalties are a large portion of the objective function value, further scrutiny of penalties is indicated. If the penalties are a small portion of the total or if their relative weight cannot be determined at this point, it may be more enlightening to examine the total budget and force structure resulting from the optimization run.

From Figure 8, the analysts sees that the O&M values, which reflect the real-world costs of Operations and Maintenance, remain relatively steady with a gradual increase towards the out-years of the run. The penalty values are high at the beginning of the model as might be expected because the model decisions have little impact on fleet condition in the earliest years. The penalties go to zero in the mid-years then become somewhat significant in the second half of the planning horizon. If the penalties were large, this would invite further analysis but because of their relatively low values the analyst may choose to first explore the budget branch.

The analyst begins with a look at the total budget expenditures of the run in relation to the minimum and maximum budget constraints in Figure 10. Recall that these constraints are elastic and incur penalties when violated. From Figure 10 the budget exceeded the maximum in the first two years then stayed within budget limits for the remainder of the run, dropping quickly in the out-years. A closer look at expenditures is therefore warranted.

Although penalties are important to the solution of the model, the actual annual expenditures are composed of only the procurement and O&M costs. Figure 11 reveals these actual expenditure figures along with minimum and maximum budget limits. The last few years of a model, in this case 2007 and 2008, may not be valid. This is a well-known aspect of such multiple-time period, finite horizon models with fixed ending and beginning conditions. It is made much more obvious by the visualization techniques used in this analysis. This anomaly is caused by the optimizer's approach to minimizing costs and incurring penalties towards the model's completion rather than investing in expensive procurement actions whose long-range pay-off would not be realized within the scope of the model. This tendency must be considered in viewing and implementing the results of the model. Closer examination of Figure 11 indicates a leveling off of procurement from the year 2000 and beyond even though annual

expenditures are further and further below the increasing maximum budget. The analyst would explore the various explanations for this trend by asking questions such as "Is the model capable of meeting all goals with less than total expenditure?" and "Is the full budgeted amount not required?".

In order to answer these questions, the analyst must first determine the answers to more basic questions such as "What did we get?" or "What did we not get?". The first question could be answered by looking at force compositions, supplemented by age and hi-tech compositions. In this instance, however, the question of what goals were not met by the solution is more instructive. It is best answered by examination of the penalty data.

Switching focus to the penalty branch of the analytical hierarchy in Figure 9, the analyst would see which goals are not being met. Figure 12 shows the amount and type of penalty assessed in each year of the model. It is understandable, but not very controllable within the model, that significant penalties are assessed in the first two model years. Their magnitude somewhat conceals the breakdown of penalties in the later years of the model which are of greater interest in the optimization process. In order to focus on the later penalties and reveal the information, the analyst can use the hypertext technique to zoom in on the years

of interest. By hiding 1989 and 1990 a new graph, Figure 13, is created.

In a similar way, at any time the analyst may see fit to focus on segments of the data that may be of significant concern or interest, for example a segment of years where goals are not met, a particular role of helicopters, etc. The graphical hierarchy and use of a hypertext technique allow the analyst to move about within the structure and zoom in on a selected period of time or other factor.

At this stage if the penalties appear to be extreme, the analyst may again question the fidelity of the model. A return to the data and problem definition may be warranted to assess the need for making changes in the penalty structure. In this case, however, the penalties do not appear extreme and the analysis process continues.

The analyst sees in Figure 13 a surge of high-technology penalties in the middle years of the model, tapering off to almost nothing. Mission requirement penalties increase significantly beginning in 1997 then decrease somewhat, and there is a steady rise in age penalties in the later years.

Since the requirements for high-technology, average age and mission requirements are different for each of the four aircraft

roles -- attack, scout, cargo and utility -- it is important to view the penalties as they effect each role. This reveals the specific areas that need further investigation. Figure 13 shows that any real trends in penalties will be found in years 1997 to 2008. The analyst again focuses on these years in the three appropriate penalty areas, Figure 14.

It appears that Scout aircraft are primarily responsible for both the mission requirements penalties, Graph 14a, and the high-technology penalties, Graph 14b, especially from 1998 to 2004. In later years, Cargo aircraft are responsible for most of the remaining penalties in the categories of mission requirements and age.

It is now necessary to look in even greater detail at each penalty category and to assimilate information from different parts of the model. The windows environment facilitates this part of the process by allowing multiple visualization tools to be displayed on the screen simultaneously. For example, in considering the mission requirement penalties, it is instructive to simultaneously view the force composition graphs and the penalty information as in Figure 15.

The analyst is particularly interested at this juncture in the Scout and Cargo helicopters. In the years where penalties were

assessed, the Scout force is composed mainly of OH-58D's, with the OH-58A's and OH-58C's having been SLEPed into 58D's. Production of LHX-SCT's begins in the model in the year 2002, at which point the build-up returns the Scout force to its minimum requirements and penalties are no longer being assessed in this category. This is all clearly visible to the analyst from Figure 15.

It is important for the analyst to examine the reasons that the optimization chose this particular course of action and to consider such questions as, "Why was the LHX-SCT not procured sooner to avoid these penalties?" and "Is it due to production limits, budget limitations, or was it driven by O&M costs in the objective function?". First, however the analysis will continue with an examination of the other force composition penalties at this same level.

The force composition of Cargo aircraft from 1997 on is primarily CH-47D's. There is no indication of production, and the model continues to accrue penalties in force composition for the duration of the model. Again, "Why are these penalties incurred and what, if any, alternatives are available?". Beginning with production limitations as a possible explanation, the analyst examines the graph of production line schedules and observes from Figure 16 that the production capability for Cargo helicopters must

close by 1994. It would not be cost effective to begin production earlier since the force structure of Cargo helicopters is adequate through 2003 and there is no production capability after 1994. This answers one chain of the single-run analysis and could be noted for additional exploration.

Returning to Figure 14, the analyst proceeds with an examination of the high-technology penalties which are due almost entirely to Scout aircraft, beginning in 1999 and tapering off dramatically at the end of the model. By examining the force structure broken down by role in Figure 15, the analyst can see that all the existing Scout aircraft switch from high to low technology in 1999. The Force Composition graph indicates that Scout force is almost entirely made up of OH-58D's. The use of hypertext would allow immediate access to the information behind the graph. The analyst would be able to view and assimilate all this information quickly and easily by retrieving these graphs onto the screen and creating an environment like Figure 17.

Once LHX production begins in 2002, the model's minimum fraction high-technology limit is almost attained for Scout helicopters by the final year of the model. The analyst may wonder what measures would have avoided the penalties. Another look at the production schedule and associated spreadsheet information shows

that LHX production could begin as early as 1995. Figure 18 shows Attack LHX helicopters were procured as early as 1997 and the production line was capacitated for the first two production years. Note that the force composition graphs do not show LHX Attack helicopters in the fleet until 1999. This is due to the lag years between purchasing and delivery. Clearly it is more cost-effective to procure Attack aircraft and pay penalties for shortfalls in the Scout fleet than to procure Scouts.

Investigation of the age penalties in Figure 14 shows the analyst that all penalties are due to Cargo aircraft, beginning in year 2004 and escalating to the end of the model. A simultaneous look at the production schedule, the maximum age and force composition graphs, Figure 19, shows that there is no production capability for Cargo helicopters, H-47's, in the later half of the

The analysis process is controlled by the user and the specific approach will depend on the results of the model and the questions to be answered. All of the input data and the results are available in the spreadsheets and graphs. The way the analyst chooses to navigate through this information is subjective. Any train of thought or process of inquiry can be followed. Once the analyst has fully explored a single run, it is then possible to begin

sensitivity analysis by comparing the solution with one or more subsequent runs of the model.

D. MULTIPLE RUN ANALYSIS

A single optimal solution is seldom acceptable for a complex optimization problem. No solution can account for all possible contingencies. As many contingencies as possible need to be considered within the realm of analysis. The developers of PHOENIX recognized this and included as one of their findings, "Mixed integer linear programming provides no useful dual information for conducting postoptimality analysis. Sensitivity analysis of model output must be accomplished using multiple runs with varying input data." (Force Systems Directorate, 1988).

In the final report on the PHOENIX model (Force Systems Directorate, 1988), two solutions were provided, each solved by the same model with the same parameters except for the budget growth rate. Fourteen other runs were made, but the results were not provided (p. 3-10). Since the goal of the study was "to formulate and implement a prototype decision aid for force planners for evaluating the effect of aviation modernization policy over an extended planning horizon" (p. 1-1), the report did not emphasize the solutions of PHOENIX but rather its role as a decision aid. The application of analysis and visualization principles presented here

would have greatly enhanced the ability of the modelers to analyze and report on these solutions. Additionally, current users of this model would better conduct sensitivity analysis by making the multiple runs and evaluating and comparing their results using this modeling strategy.

This section will discuss two ways to perform multiple run analysis and apply each to the PHOENIX model. The first scenario, which is probably the more common, solves a baseline model with the best-known parameters. The analyst may want to explore how relaxations or changes to these parameters effect the solution. This is typical in many optimization situations, especially in ones like PHOENIX that are time sequence problems with multiple measures of effectiveness in the objective function and constraints.

The second approach also involves the changing of parameters, however it does not presume that one set best represents the solution. Rather it may compare a variety of solutions obtained by systematically changing one or several parameters, such as inflation rate or budget growth rate. Although this approach was not originally reported in the PHOENIX study, it is practical and useful, especially when considering a long term planning process in an era of ever-changing government spending priorities. Many of the

same graphical techniques can be applied to both cases although they may be more instructive for one purpose over another.

1. General Comparisons

One of the most important aspects of comparisons of solutions is the comparison and documentation of the parameters that change. Since the input data for each run of the model is displayed in graphs or tables, it is relatively simple to produce side-by-side comparisons of these values. Highlighting the information that is different is also accomplished very easily in these environments. Figure 21 demonstrates, for a two case scenario, how effective the side-by-side comparison can be in both graphical and tabular form. Figure 21a shows the different objective function values and their breakdown into O&M costs and penalties. The relative contributions of each of these components as well as the actual objective function values is quite apparent. Figure 21b, on the other hand, shows the change in the purchasing cost of LHX aircraft, a change that can drastically alter the outcome of the solution, displayed in spreadsheet form.

2. Comparison to Base Solution

The objective function value is typically one of the most important criteria for comparison of solutions. However, this is not always the best criteria. In a model like PHOENIX which has

elastic constraints and associated penalties that are incorporated as part of the objective function value, careful consideration must also be given to the rest of the solution. It is important to note which goals have been met and which have not been met in each solution over the time period of the model. It is also important to look at the differences between each of the models for each of the constraints. For example, Figure 22 shows a comparison of the total annual expenditures of two runs of the model. The line graph, Figure 22a, indicates the actual expenditures in the two cases but Figure 22b is more enlightening as it is a graph of the differences in the two solutions. It clearly shows that the expenditures of the second run are consistently lower than those of the base case. These same types of graphs can be constructed for force compositions, procurements and the other constraint conditions. They can also be used for determining the differences in the penalties that are assessed in each situation.

Another type of graph that can be used to compare results of two runs is the scatterplot depicted in Figure 23, which displays the same information as Figure 22a. Clearly, any deviations from the identity, or x=y, line indicate differences in the two solutions. A majority of the points to the right of or below the x=y line, as in Figure 23, indicates larger values for the solution

associated with the horizontal axis, in this case for the base solution. If points were scattered on either side of the line, neither solution would be generally better or worse.

When comparing several different solutions against a base case by display in a small multiples format, these graphs become quite dramatic and informative. The eye is able to quickly make comparisons and

inadequate solutions may sometimes be discarded.

3. Comparison of Several Solutions

The use of multiples described above also works very well in the case of the comparison of several solutions. This is demonstrated in Figures 24 and 25. Figure 24 is a multiple display of the objective function values, broken into O&M costs and penalties. The eye can focus on the differences in the data and the mind is able to quickly assimilate this information. It is much more effective than having to turn pages or even look at two pages simultaneously. Figure 25 is an even more striking display of the objective function values. While it does not give quite as much information as Figure 24, the boldness of the display and the close proximity of the graphs to each other makes assessments of their relative values very apparent.

Other data analysis techniques can also be used to compare several solutions. The use of color serves very well in distinguishing between several solutions. A single color associated with each particular solution lends consistency to the design and can serve to highlight the best solution or at least narrow down the field to a few solutions that can be further analyzed. Even when color is not available, the use of different shades of gray or different textures serve the same purpose, as shown in the PHOENIX application.

When dealing with many possible solutions, it is instructive to use some statistical tools. For example, when discussing the force composition of attack aircraft in each year of the model, it may be useful to consider the minimum, maximum and average number of aircraft for each year from all the runs of the model. This could be done quite efficiently with boxplots, Figure 26. These plots can indicate the distribution of the data and show any skewing. These boxplots could be used to analyze constraints. If a display of the results of multiple runs of the PHOENIX model, for example the number of attack aircraft of a certain type procured in each year, showed little variation, this could indicate to the analyst that this constraint might be tightened or restricted in some way without effecting the rest of the model.

E. PRINCIPLES APPLIED TO PHOENIX

The graphs and spreadsheets of the previous sections illustrate the application of particular analysis and visualization principles to the PHOENIX model. There are, however, some general principles that apply to all the graphs.

One of the goals of visualization is to reveal the information contained in the data. The eye should focus on the information of interest. The graphs are created to bring any outlying values, major discrepancies or trends into the forefront.

Revealing constraint violations is also important. This is done by combining bar and line charts. The actual values that the solution produces, such as the force composition of attack aircraft, are of primary interest, but the analyst also needs to see how this number compares with its goal. One piece of information complements the other but does not overpower it as it can easily be ignored if not of immediate interest.

Some information is duplicated in a single graph. This is the case in Figure 11, the annual expenditures. Both the height of the bar and the text at the top of the bar contain the same data. This was done because of both the magnitude of the information and its importance to the overall solution.

Another valuable aspect of these graphs, and one of the visualization principles, is consistency. Graphs of comparable values are consistent in size, coloring and labeling. The order of aircraft, both individually and by role, also stays the same. This consistency is important to the analyst because it reduces the amount of time spent interpreting the data in favor of time spent analyzing it. It also facilitates combining similar information, such as the number of Scout and Attack aircraft procured in a year, which lends a different view to the analysis process.

Finally, the graphs are designed to highlight and explore the relationships among the data values. This is accomplished through the windowing environment which allows the overlaying of one graph upon or next to another. The relationships can also be explored by the display of data values themselves. These values are always available in easily accessible spreadsheets. The hypertext capability will ensure these are available at the click of a button.

There are also other features not currently included in this analysis due to limited technology but which could be developed to be of great assistance. One technique is to capture the thought processes of the analyst in a scripted scenario that could later be reviewed. This could be of significance when a model is only used periodically. If the analyst decides to update or review the plan,

it would be invaluable to be able to see the process by which analysis was initially conducted and how the decisions were made when the model was originally run. In many circumstances a new analyst might be updating the plan. That analyst could follow the scripted version of the initial run and gain much insight and understanding prior to conducting follow-up runs and making changes.

This scripted version could be designed to lead the analyst through the input data and results in the same order as was originally done. Voice narration could even be included to further document the decisions. The graphs could appear in a flip-chart format; they could fade in and out or they could be programmed to open and close as appropriate. This script could also be used by the analyst to present decisions or options to planners.

VII. THE FUTURE OF PHOENIX

When it was first implemented in early 1988 PHOENIX provided the Army with a detailed plan for the Army Aviation Modernization Program (AAMP. This plan was revealed in a press release by the Secretary of the Army dated September 30, 1988 (Brown et al, 1991) which stated:

> The funding provides for an efficient, cost-effective production rate of...aircraft in quantities required by the Army's force structure in meeting the requirements of the unified and specified commanders-in-chief, and to achieve an optimum program within the funding constraints.

In essence, PHOENIX has guaranteed a modern fleet of helicopters within budget that will carry the Army through the next 25 years and beyond. Since that time, PHOENIX has also been successfully modified and adopted for use in the force planning of tactical wheeled vehicles. Additionally, PHOENIX has been expanded to include production of component parts and subassemblies of major systems as well as transfers from one force package to another. A force package is a collection of units grouped together based on deployment schedules such as active versus reserve forces. (Coblentz, 1991)

In addition to PHOENIX, CAA uses another force planning tool, the Force Modernization Analyzer (FOMOA). It is a scaled-down non-integer linear program version of PHOENIX. It is implemented in spreadsheet form on a Macintosh personal computer using the Super MacVino optimization package. It is designed for use as a quick reaction analysis tool and has a turnaround time of approximately twenty minutes.

These two force planning tools are used to complement one another. PHOENIX is the far more capable model but it requires well-trained analysts. FOMOA does not require any mathematical programming skills but does require user input of such decisions as which production lines will open. The constraints of FOMOA are a subset of the constraints in PHOENIX and cannot be violated. The combination of time, data resources and level of detail required dictate which model is used in each circumstance. (Coblentz, 1991)

PHOENIX requires significantly more input data and its results require more extensive analysis than FOMOA. This is one of the inherent differences between the two models and cannot be changed without changing their purposes. One of the primary advantages of FOMOA is its quick turnaround time. PHOENIX originally required approximately 10-12 minutes of computer processing (CPU) time for a single run, but now runs in one to two minutes. However, CAA

does not currently have the computer capability to support a run of PHOENIX so it must be run on a computer at the Pentagon. This adds considerable time to the run of the model, not only physical commute time, but also time spent waiting for processing by the Pentagon computer.

The scheduled addition of a workstation to CAA headquarters will give them the computer capability to solve the PHOENIX model. While it takes more CPU time on a workstation than on a mainframe, the total time required to conduct a run of PHOENIX should decrease as it would no longer have to compete with other Pentagon projects for processing and the commute will be eliminated. The addition of an in-house workstation will reduce the time to solve PHOENIX from days to minutes.

Application of the analysis and visualization principles to the PHOENIX model as demonstrated in the previous chapter would similarly reduce the time required to conduct a full run of the PHOENIX model, from data validation to multiple run analysis. After the results of a solution run are entered into the spreadsheet, all the worksheets and graphs described in the previous chapter are constructed in a matter of seconds.

This combination of a workstation environment and the modeling strategy would bring some of the convenience of FOMOA to the

sophistication of PHOENIX. Problems that require the details of the PHOENIX model would be solved faster and more efficiently. It would allow more thorough analysis of multiple runs in a shorter time and would allow more runs to be made in order to explore a greater number of alternate optimal solutions.

VIII. CONCLUSIONS

The widespread availability and technological advances in computer capability and the power of information in today's society support the steady growth of mathematical programming and optimization in business, industry, government, and academia. Today's real, complex and important problems can be solved through the use of large-scale optimization with a fidelity and accuracy that was impossible a few years ago. The management and economic impact of using operations research is now widely recognized and the tools and techniques are being implemented extensively. As bigger problems are modeled, the voluminous input and output increase the need for effective analysis while complicating its execution, frequently making the time and abilities of the analyst the limiting factor in the process. The ability to make changes, to update and explore the possibilities of the model, and to communicate the results to the people who have the problem is time consuming and difficult, but crucial to effective analysis. Through implementation of the modeling strategy developed in this thesis, these obstacles can be overcome and thorough, effective validation and analysis of large-scale optimizations can become the norm.

The principles of analysis and visualization outlined as the basis of this modeling strategy are tailored to a specific problem to produce the hierarchical structure of customized tools, such as graphs and spreadsheets. The analyst controls the process and focuses on areas of interest. The application of appropriate displays, simple graphical techniques, representation driven graphs, and consistency between representations allows the analyst to more quickly perceive the extensive information and to readily identify trends, outliers, constraint violations, and penalties accrued on elastic constraints. The analyst can then relax constraints, change penalties, and explore a variety of feasible and alternate optimal solutions. The access to source data and relationships among values and the backward compatibility of this modeling strategy facilitate this analysis.

Having developed the structure of validation and analysis tools for the particular problem, data from subsequent runs of the optimization can be ported into spreadsheets and the same analysis tools created for this data in a matter of seconds. Multiple representations can add new views or information to extend the model. Sensitivity analysis and comparisons of multiple runs are significantly enhanced by this strategy and by the use of side-by-side comparisons, base case comparisons, and multiples.

This modeling strategy is effectively applied to large-scale optimizations in general but is especially expedient for multi-year and recurring models and the complex and subjective problems more frequently modeled today. The benefits of creating the tools and implementing the strategy are quickly realized. The developed model is easily learned by second-generation and subsequent users and is easily reviewed between uses.

As large-scale optimization models play an increasing role in industry and government, analysts will seek better ways to represent and solve complex problems. In cases where an analyst would otherwise not be able to thoroughly assess voluminous data and a large array of alternate solutions, implementation of this strategy will support thorough analysis and selection of the best possible decision.

APPENDIX

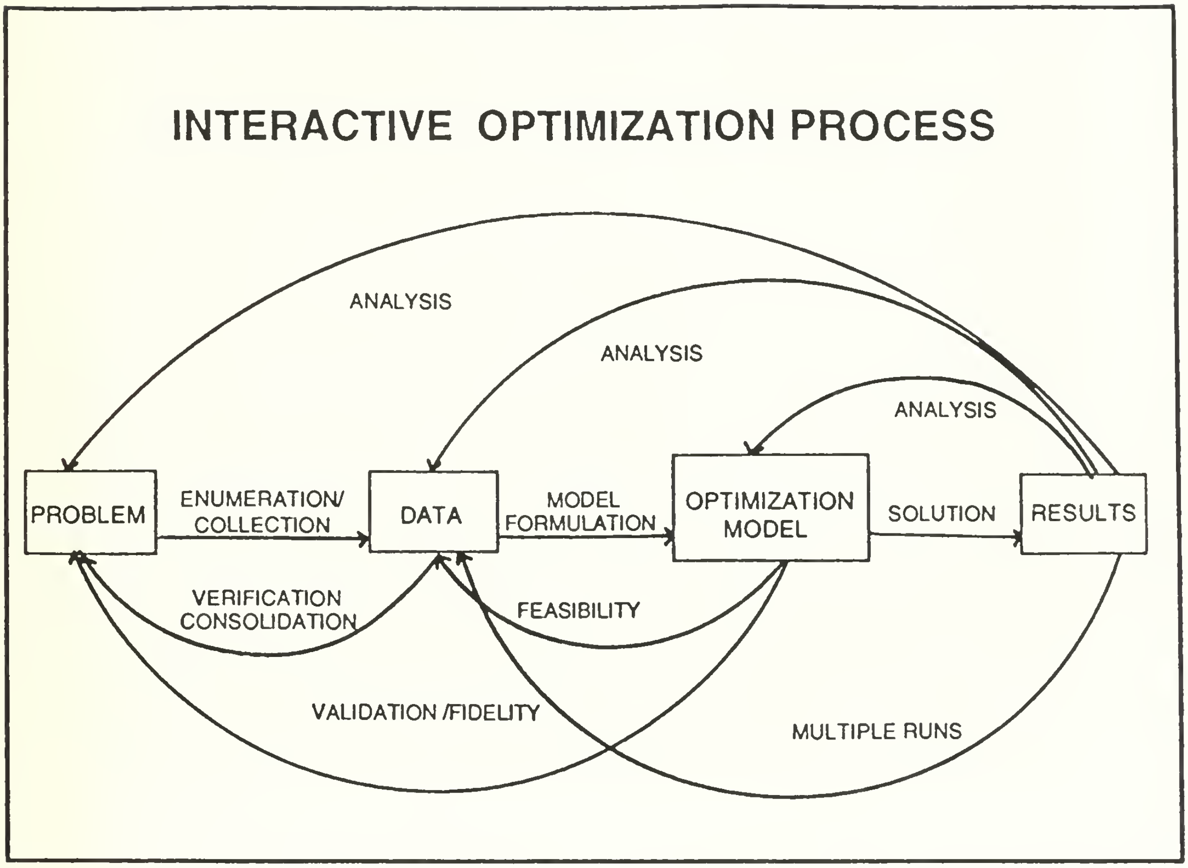


Figure 1. Interactive Optimization Process. The traditional optimization process is indicated by blocks and forward arrows. The modeling strategy for the interactive optimization process emphasizes the analysis, verification/consolidation, validation/fidelity and multiple run arcs.

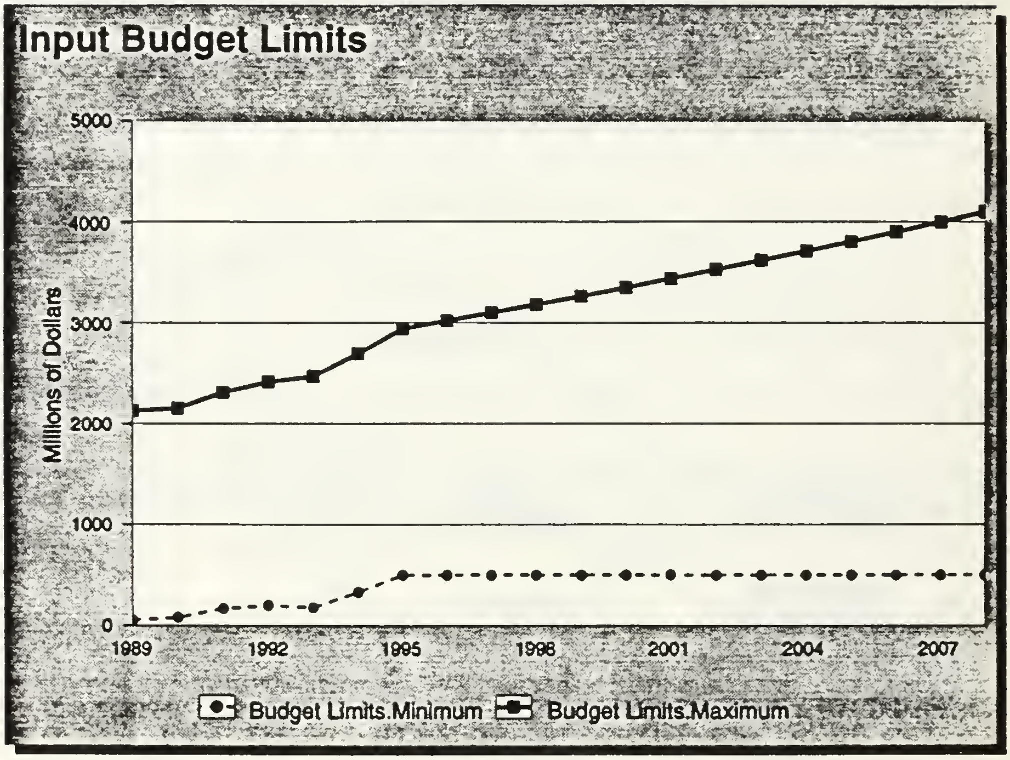


Figure 2. Budget Constraints. Verification of input values is more readily conducted through the use of presentation graphics. As expected, the maximum budget input in this case is steadily increasing and the minimum budget input levels off after stabilization of the model. This type of graph supports the visualization principles of simplicity and appropriate displays.

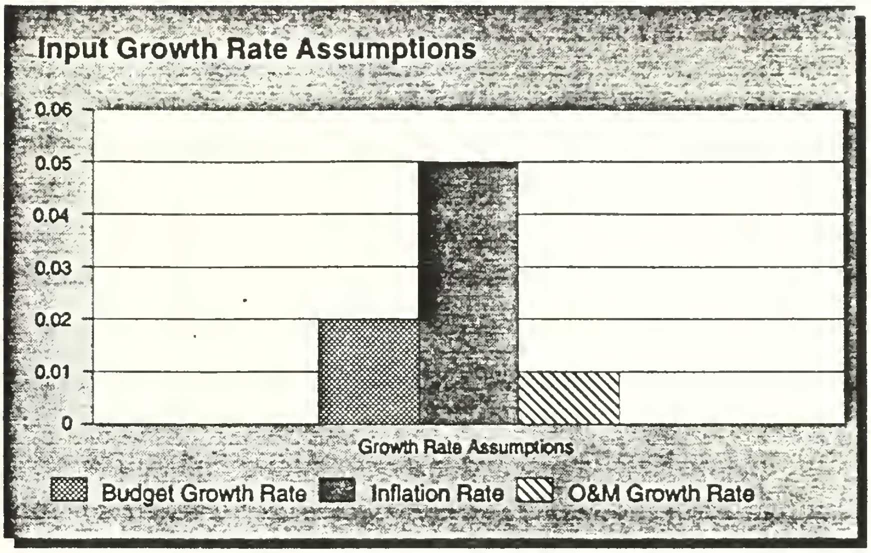
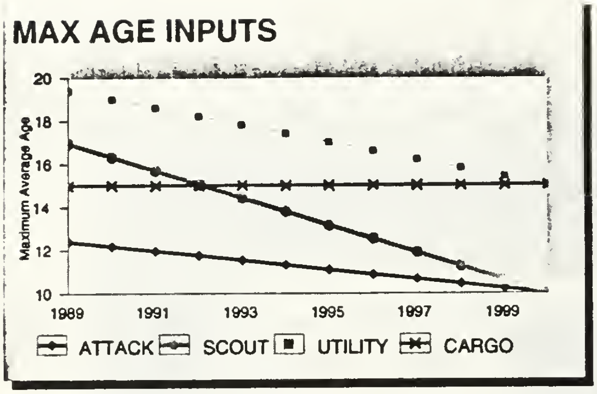
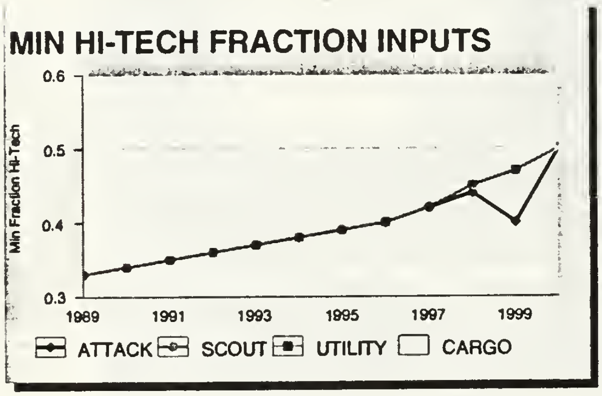


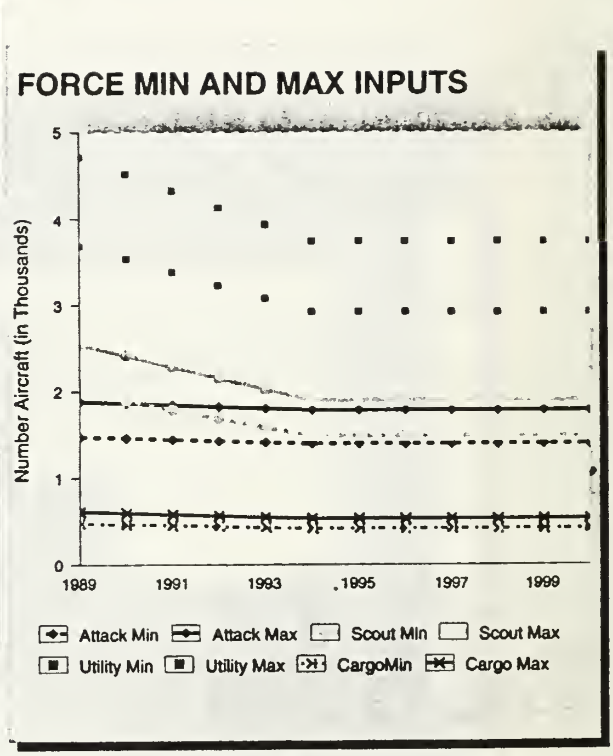
Figure 3. Parameter Growth Rates. Input parameters are easily compared using a simple bar graph. The visualization principle of simplicity is highlighted here.



4(a)



4(b)



4(c)

Figure 4. Policy Parameter Input by Helicopter Role. Minimums and maximums should appear as increasing or decreasing functions. The drop in Figure 4(b) in 1999 alerts the analyst to a possible input error for Attack helicopters. The visualization principle of consistency is portrayed by the use of identical line types for type of helicopter in each graph, facilitating interpretation.

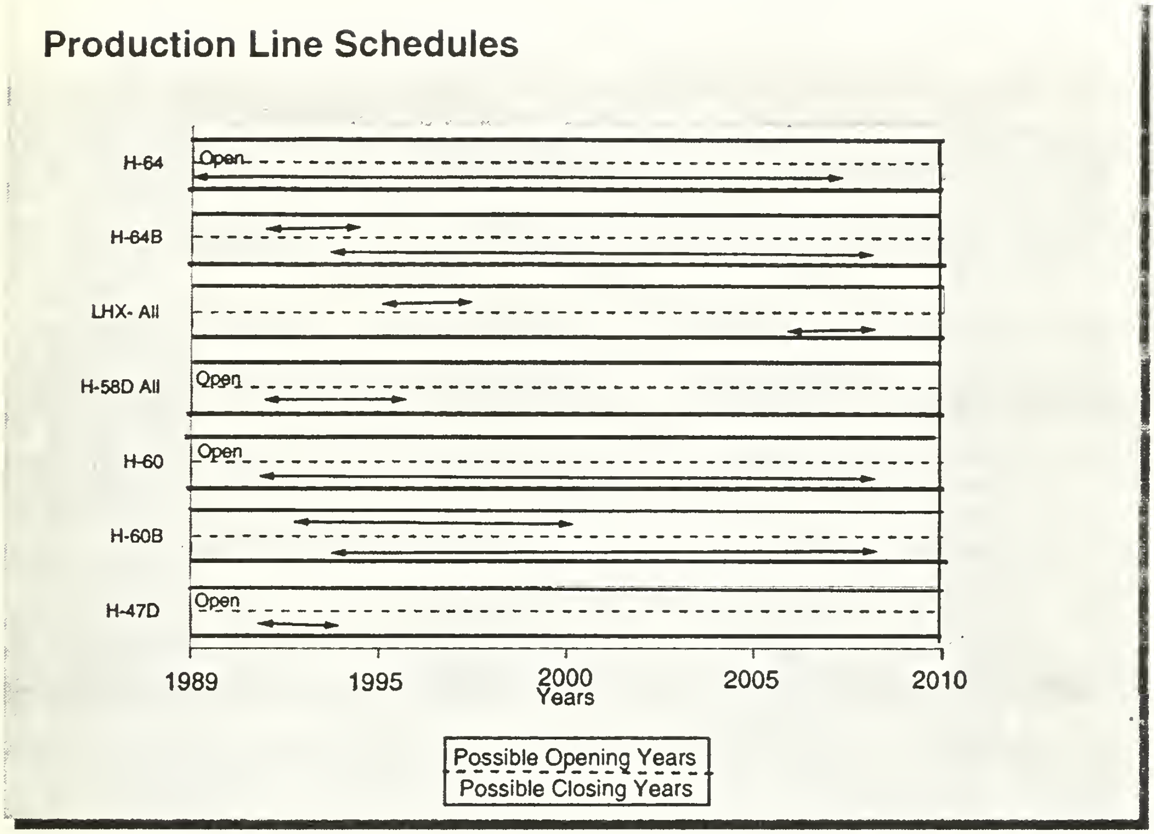


Figure 5. Schedule of Production Lines. This graph is useful in the validation of production line input data. It also will be useful in the analysis of the model to find causes of penalties and force composition shortfalls. It demonstrates the visualization principle of data access.

		Prod Line	Last Yr Hi-Tech	Max Age	Yrly Attr %	Lag	Efficiency	Purchase Cost
Attack	AH-1S		0	25	0	1	1	0
	AH-58D	H-58D All	1998	20	0	1	1	5.54
	AH-64	H-64	2001	20	0	2	1	13.15
	AH-64B	H-64B	2008	20	0	2	1	14.65
	LHX-ATK	LHX-All	2008	20	0	2	1	7.5
Scout	OH-6		0	25	0	1	1	0
	OH-58A		0	25	0	1	1	0
	OH-58C		0	25	0	1	1	0
	OH-58D	H-58D All	1998	20	0	1	1	5.23
	LHX-SCT	LHX-All	2008	20	0	2	1	7.5
Utility	UH-1H		0	30	0	1	1	0
	EH-1		0	30	0	1	1	0
	UH-60	H-60	2006	30	0	1	1	5.01
	UH-60B	H-60B	2008	30	0	1	1	6.51
Cargo	CH-47A		0	30	0	1	0	0
	CH-47B		0	30	0	1	1	0
	CH-47C		0	30	0	1	1	0
	CH-47D	H-47D	2008	30	0	1	1	5.09
	CH-54A		0	30	0	1	1	0
	CH-54B		0	30	0	1	1	0

Figure 6. Segment of Spreadsheet. Verification of sparse data and isolated values is easily conducted in the spreadsheet environment. This supports the visualization principles of simplicity and appropriate displays.

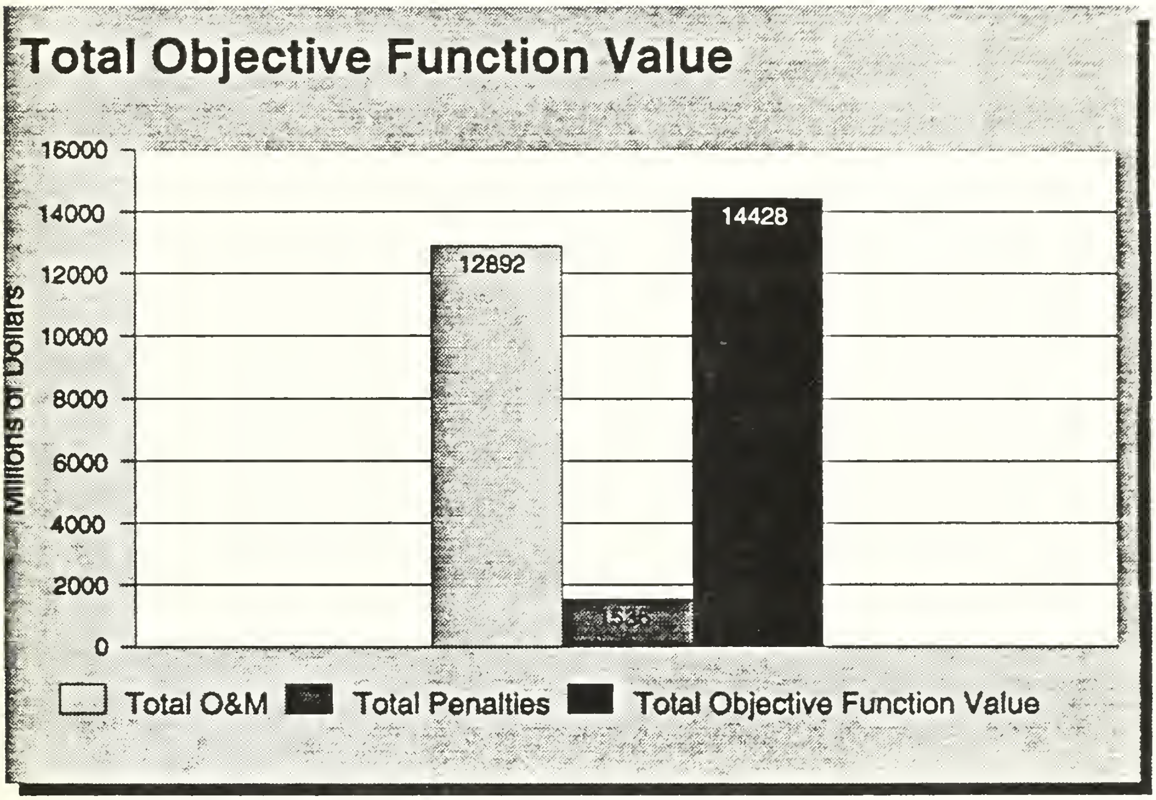


Figure 7. Total Objective Function Value and Components. This graph shows the beginning of the analysis principle of hierarchical structure. The objective function value is comprised of the O&M costs and the penalties.

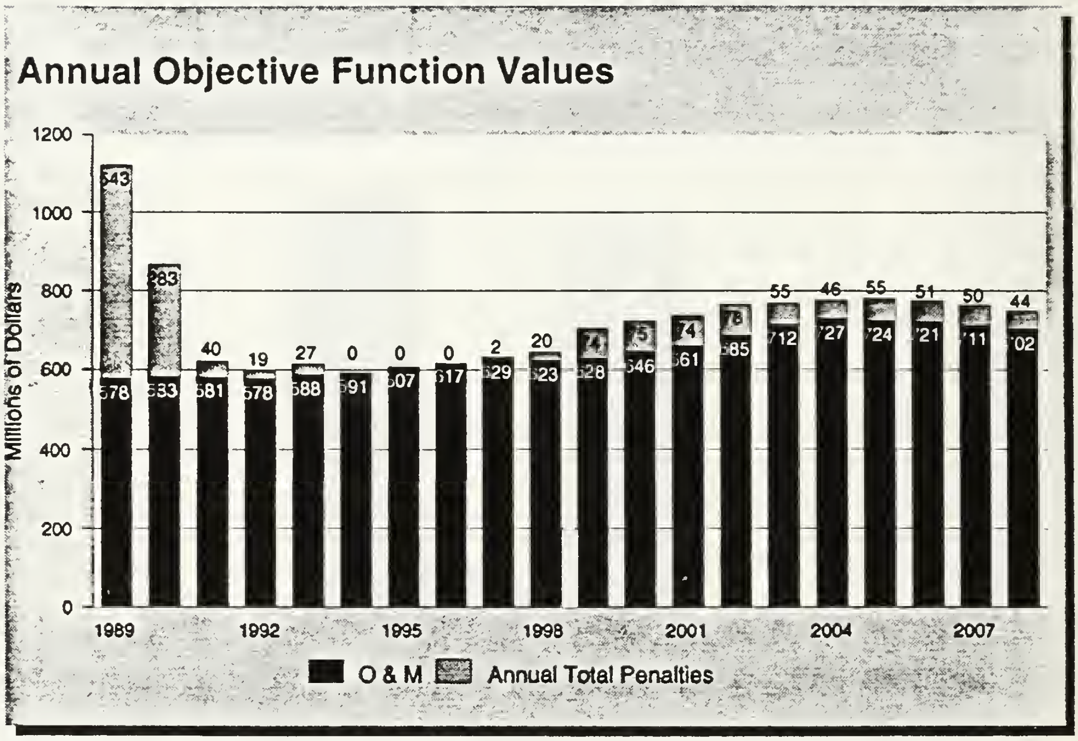


Figure 8. Annual Objective Function Values and Components. By depicting annual components of the objective function, this graph is a different level within the hierarchical structure. It portrays the visualization principle of representation driven graphs.

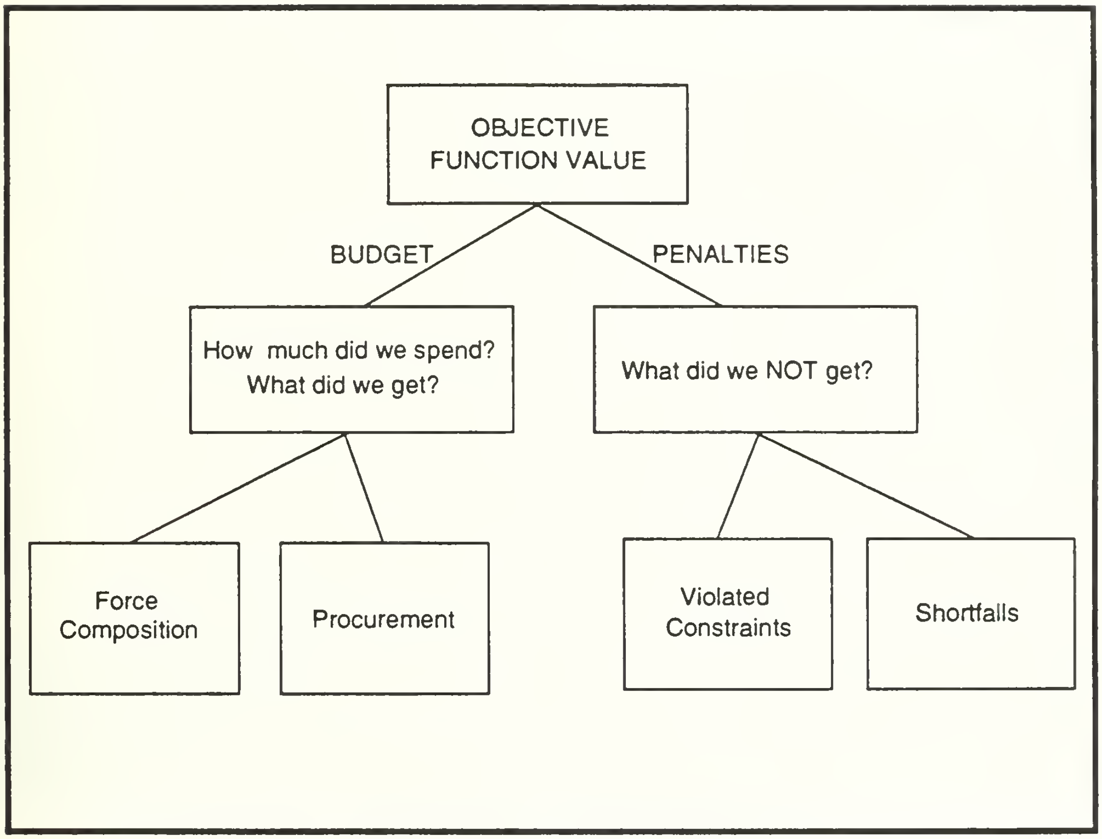


Figure 9. Hierarchical Structure of PHOENIX. A hierarchical structure is common to the modeling strategy for large-scale optimizations. The specific elements of the structure would vary by specific application.

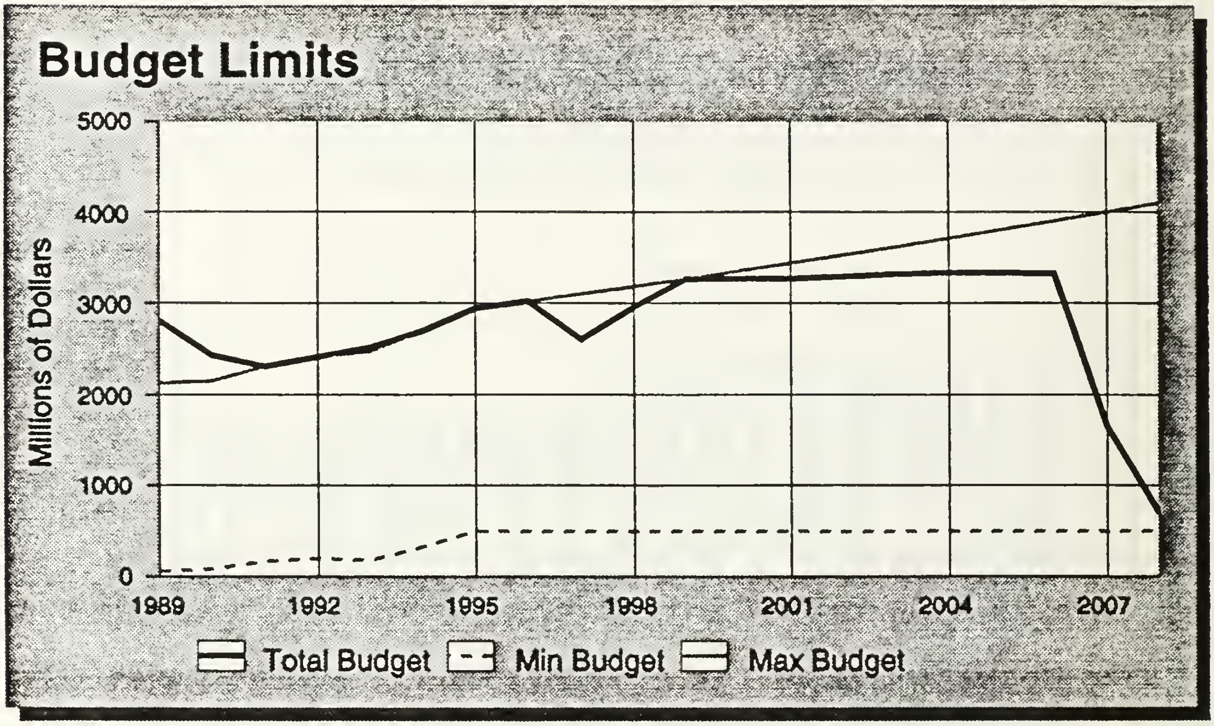


Figure 10. Budget Constraint Violations. The simplicity of the line graph makes it easy to see where the optimal annual budget violated budget minimums or maximums. This graph also pursues the budget branch of the hierarchical structure and represents the visualization principle of representation driven graphs.

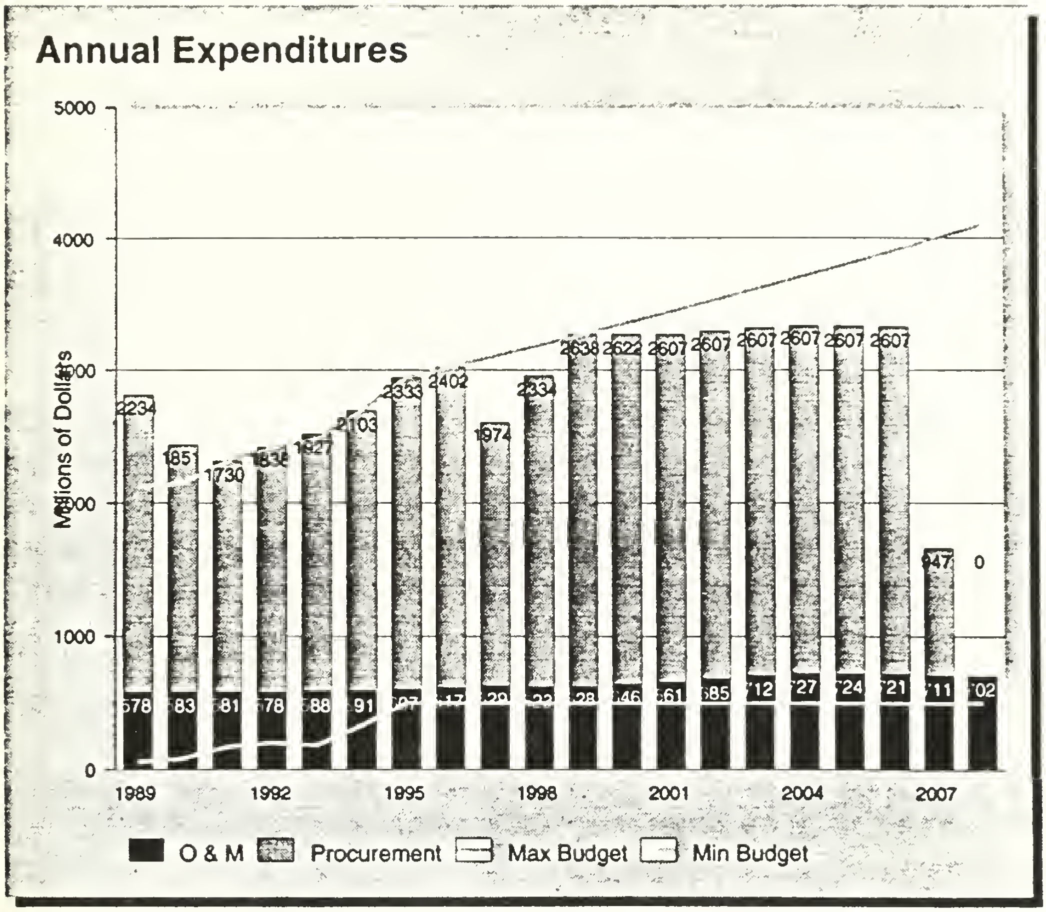


Figure 11. Annual Expenditures by Component. This graph continues in the hierarchical structure and portrays the visualization principles of representation driven graphs, appropriate displays, and simplicity.

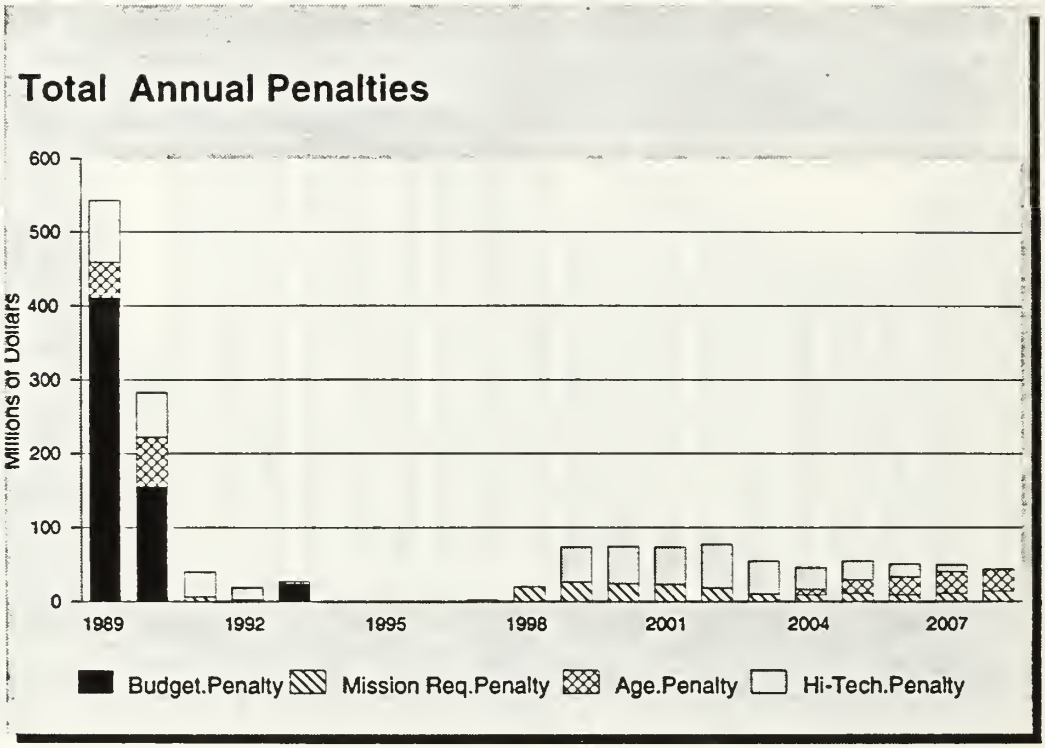


Figure 12. Total Annual Penalties. Analysis within the hierarchical structure can proceed on a different branch, as demonstrated by switching from budget data to penalty data.

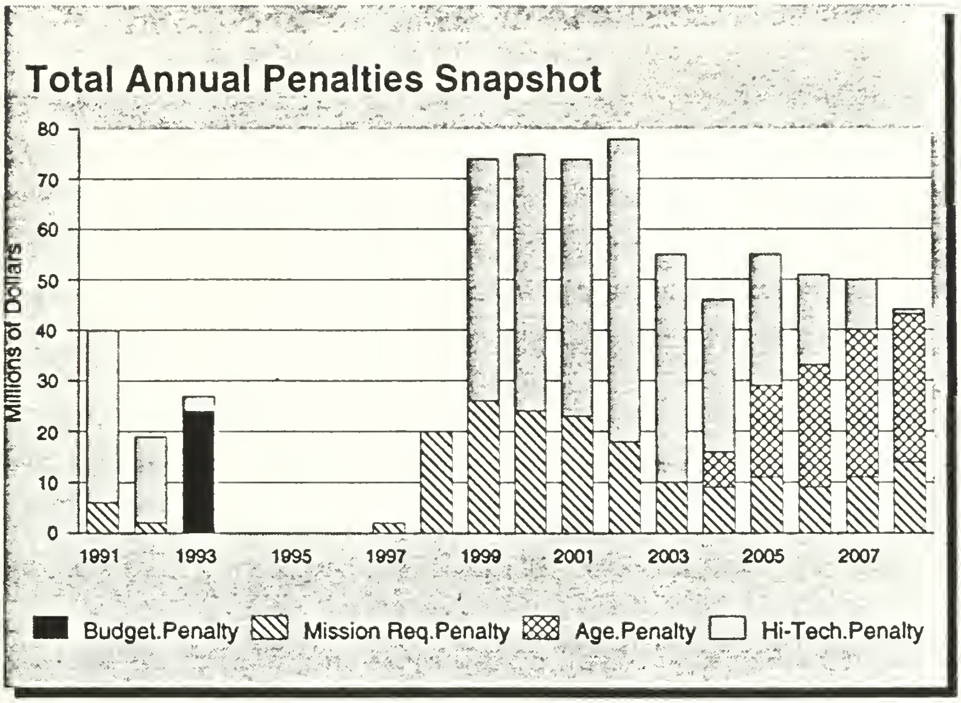


Figure 13. Penalties components for selected years. The use of the zooming principle of visualization allows the analyst to view in greater detail the penalty values for selected years of interest. This graph also highlights the principles of hierarchical structure, consistency and appropriate displays.

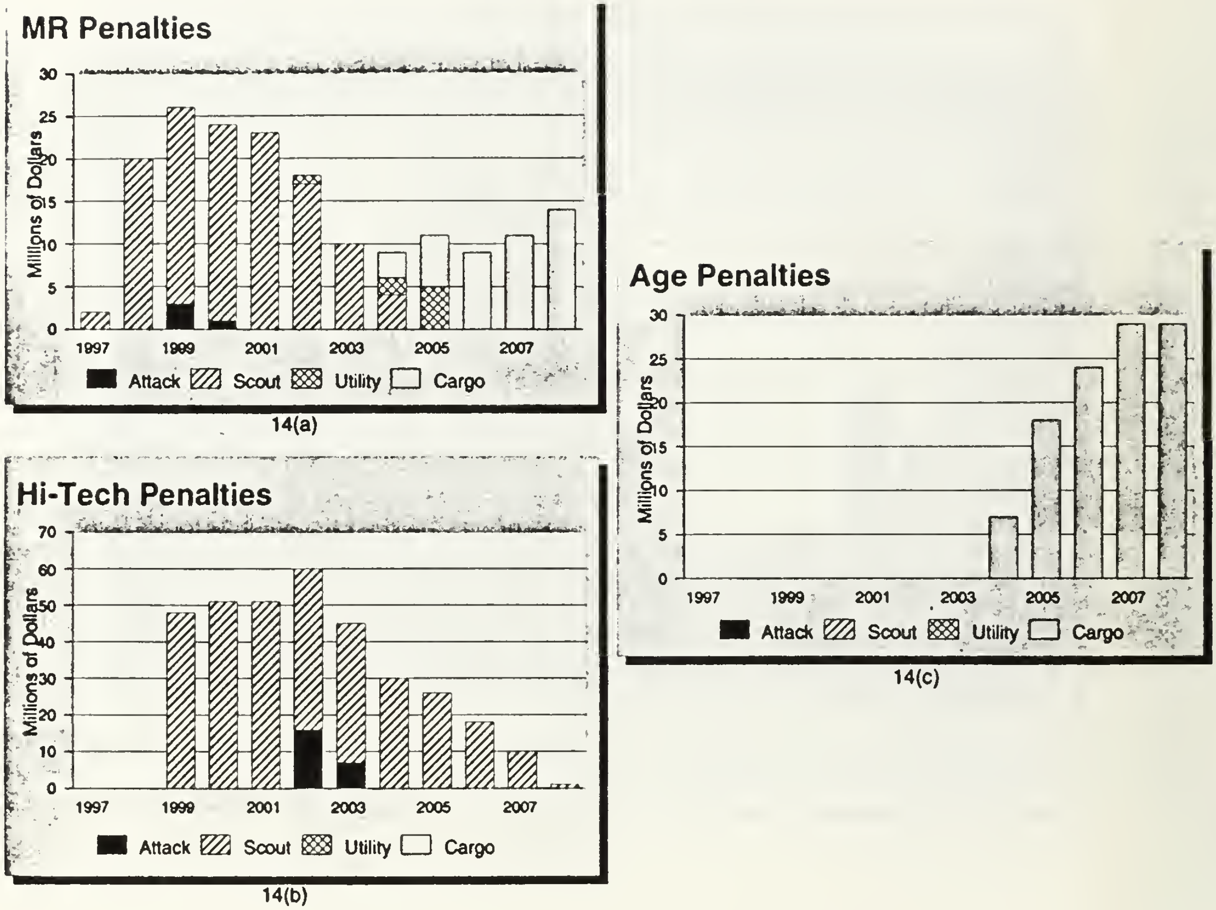


Figure 14. Type of Penalty Incurred by Aircraft Role. The hierarchical structure aids in the identification of specific penalties incurred and their relative amounts. The use of the zooming technique focuses the portrayal of only those years where penalty values were significant to the model. This set of graphs also demonstrates consistency, simplicity and appropriate displays.

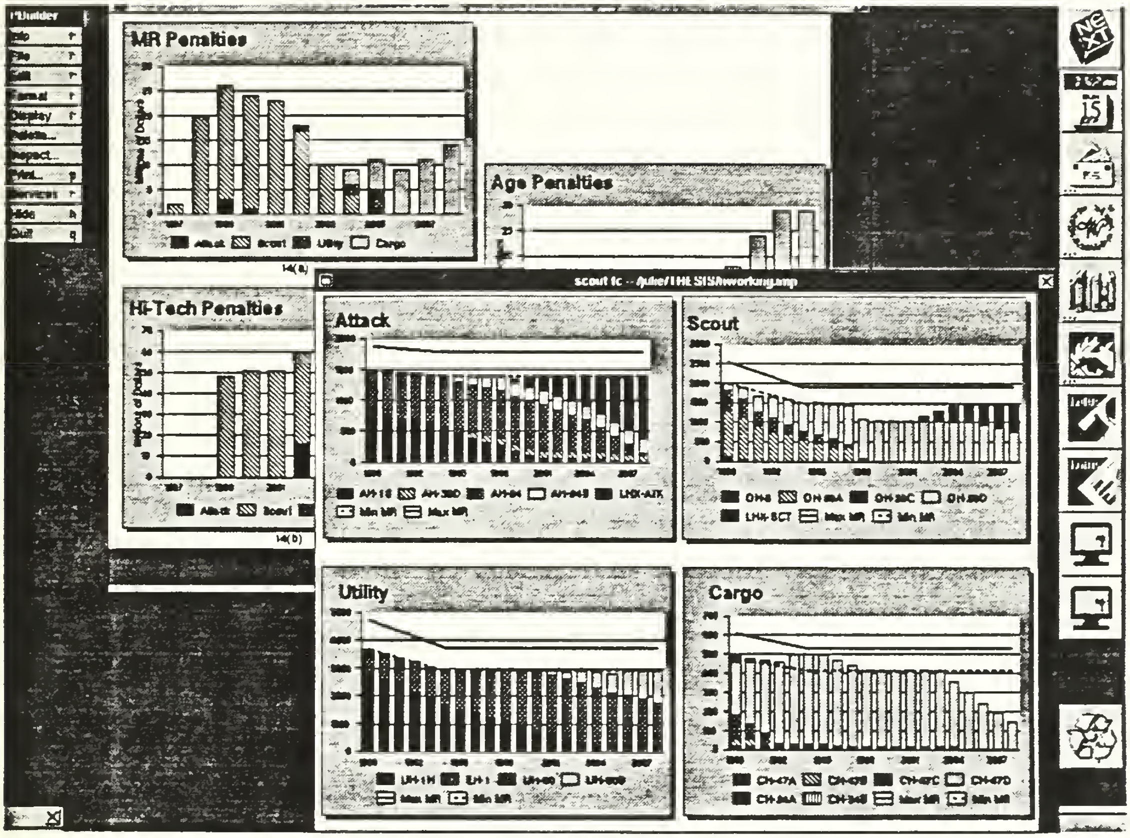


Figure 15. Application of Windowing Environment. This allows for efficient on-screen comparisons of additional views of information within the hierarchy. The combination of bar and line charts (front) helps the analyst quickly spot violations of constraints. This is an example of a side-by-side comparison.

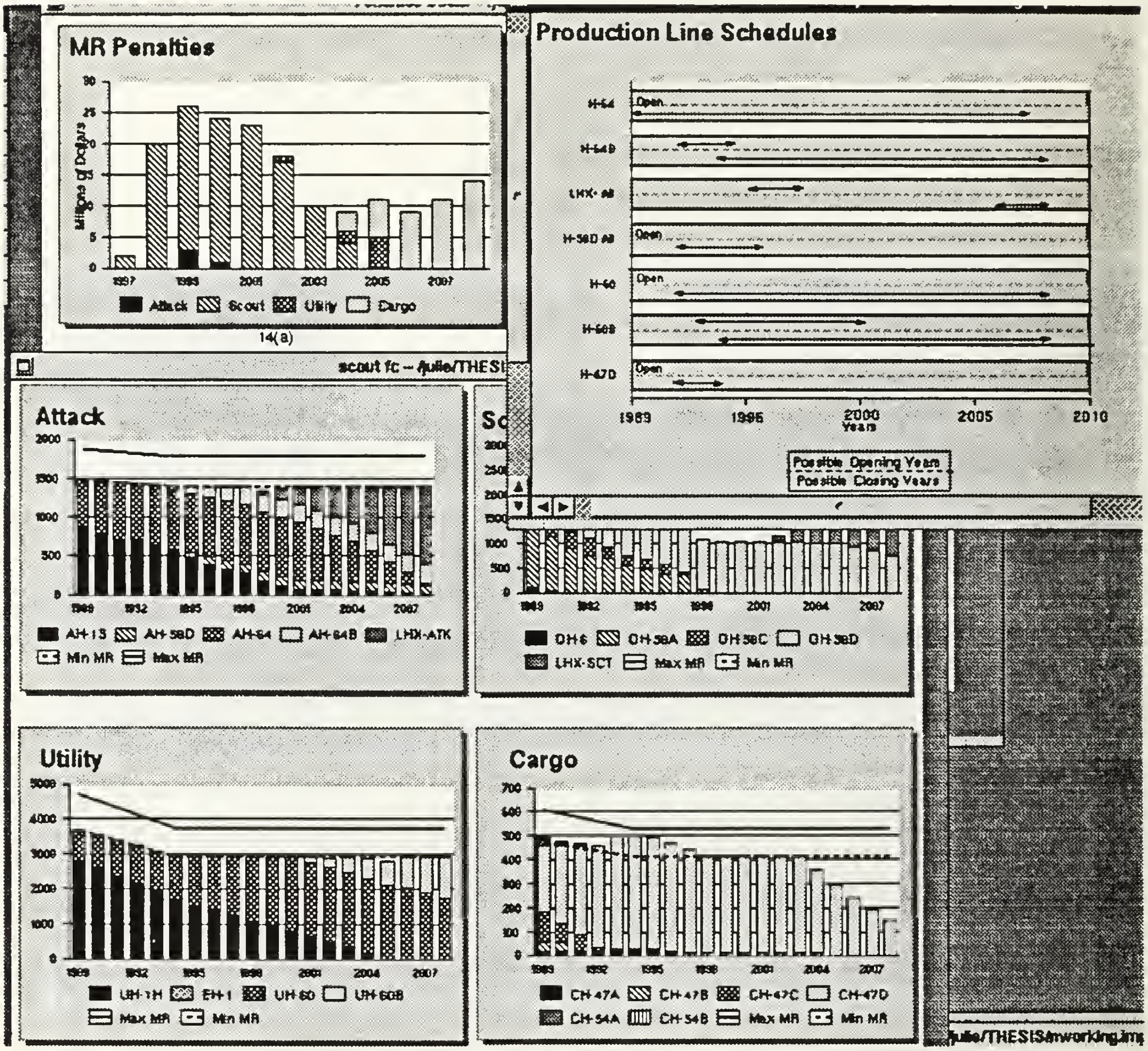


Figure 16. Application of Windowing Environment. This view supplements the on-screen comparisons in Figure 15 with the Production Line Schedules shown in Figure 5 to enable the analyst to identify potential causes of the Mission Requirement constraint violations for specific aircraft models. The side-by-side comparisons and the user controlled environment are key principles at this stage of the analysis.

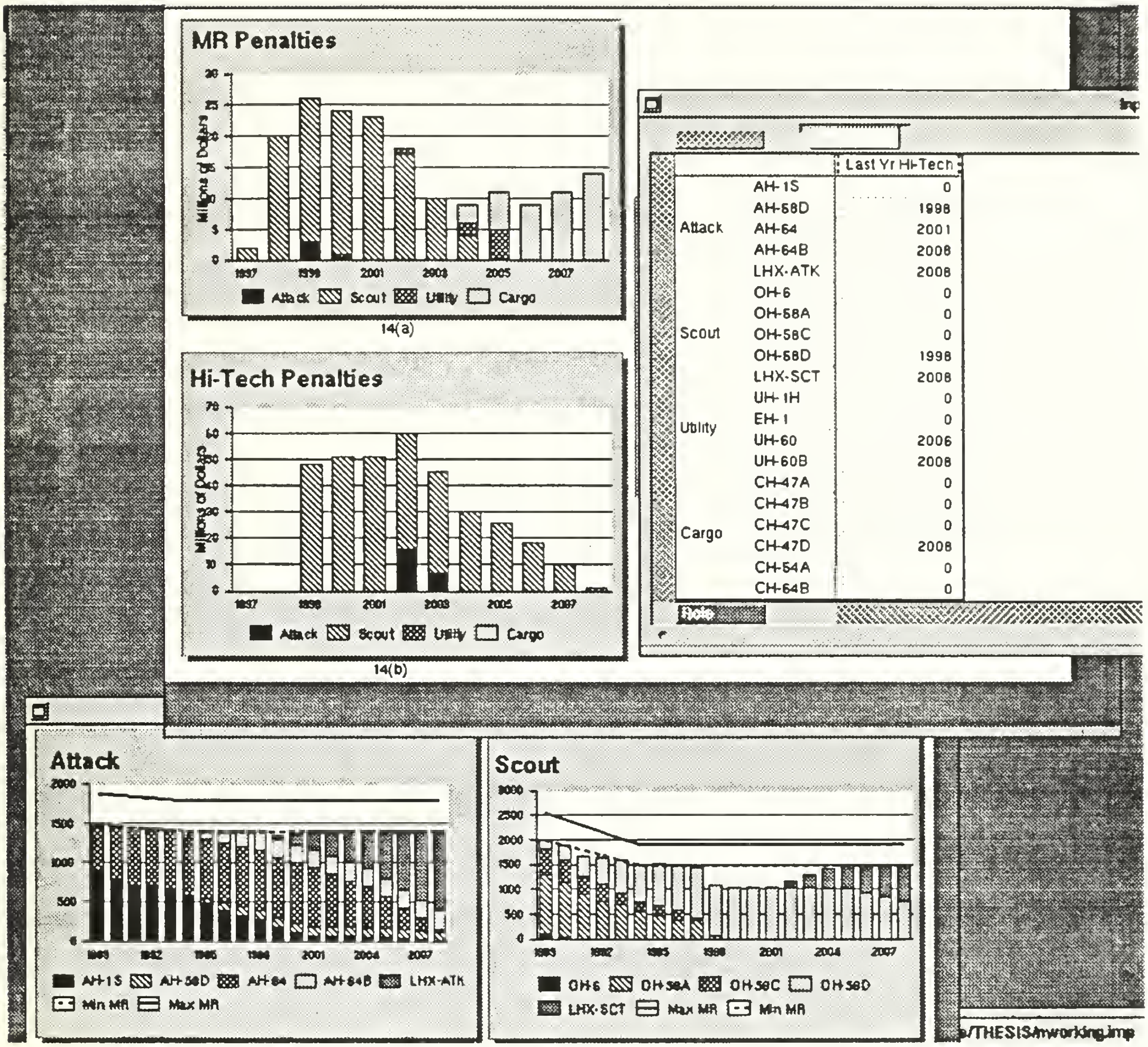


Figure 17. Windowing Environment with Spreadsheet. The user control and data access principles make it easy for the analyst to view the figures in the appropriate spreadsheet that support an aspect of a graph under investigation. The windowing environment supports the side-by-side comparison of this information.

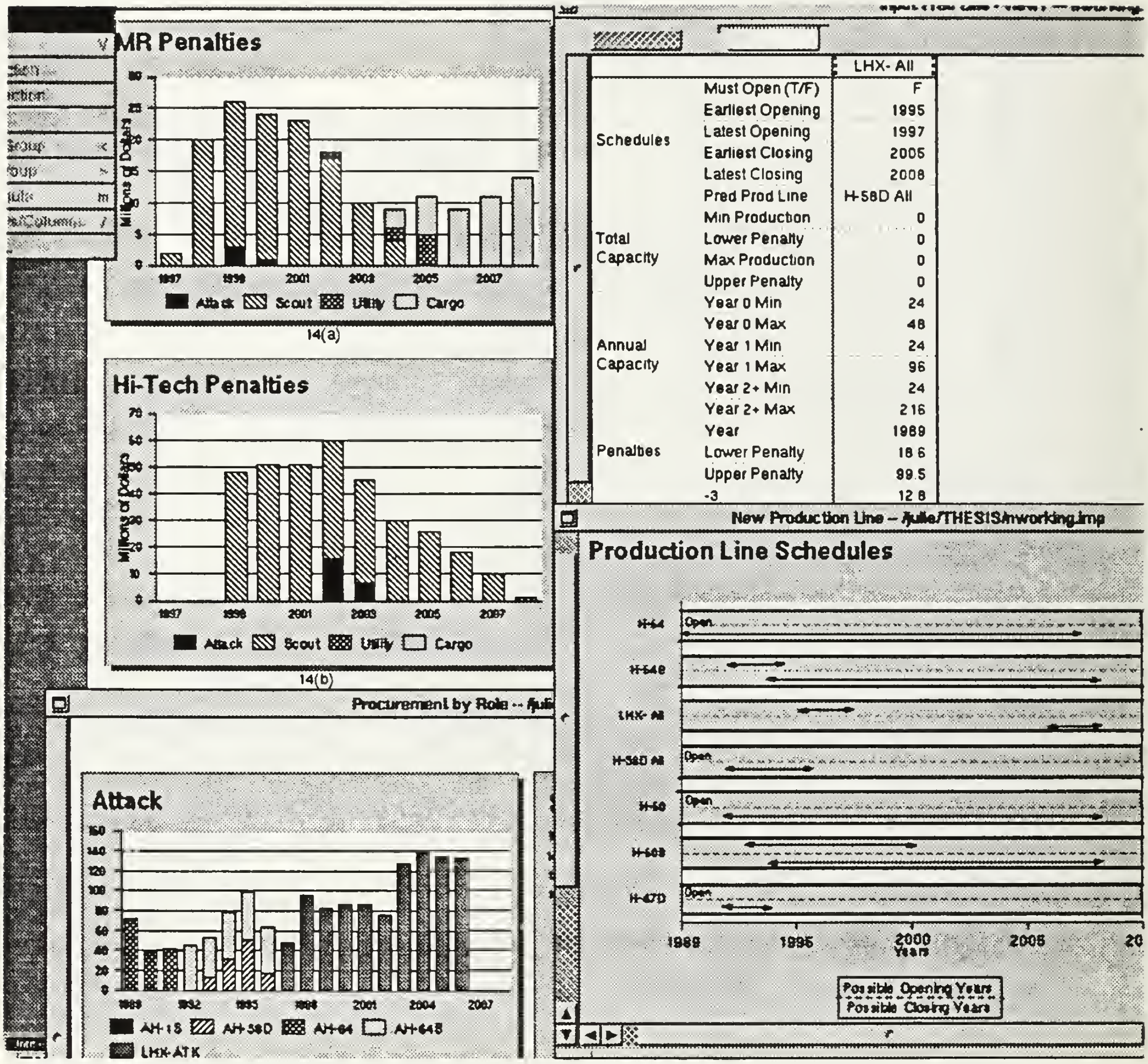


Figure 18. Windowing Environment for Side-By-Side Comparisons. The on-screen combination of a spreadsheet and two types of graphs demonstrates analytical advantages. It highlights the principles of user control, data access and appropriate displays.

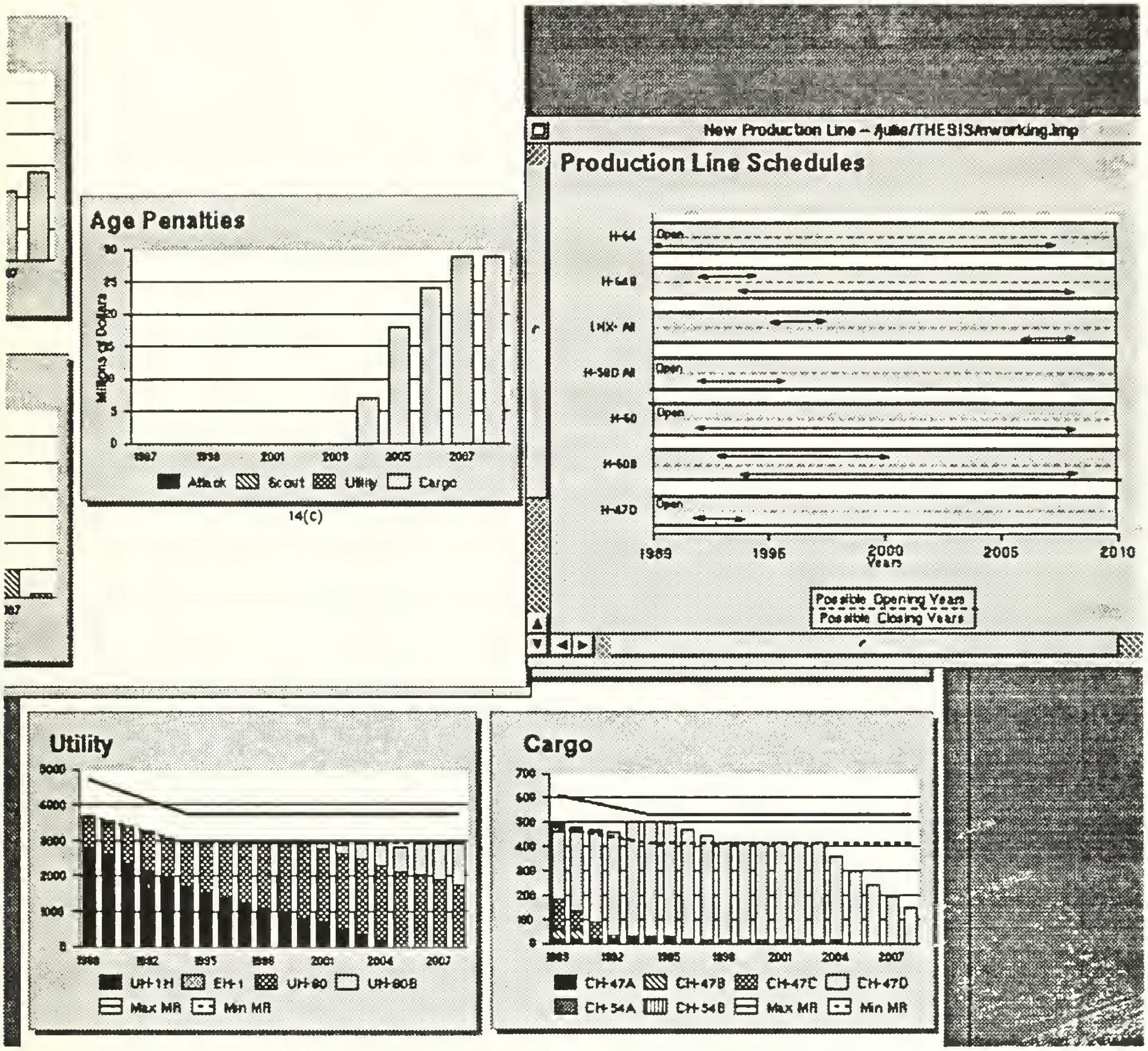


Figure 19. Continued Application of Windowing Environment. Approaching the lowest level of one branch of the hierarchical structure, the analyst can use the side-by-side comparisons to answer questions about the adequacy of the solutions.

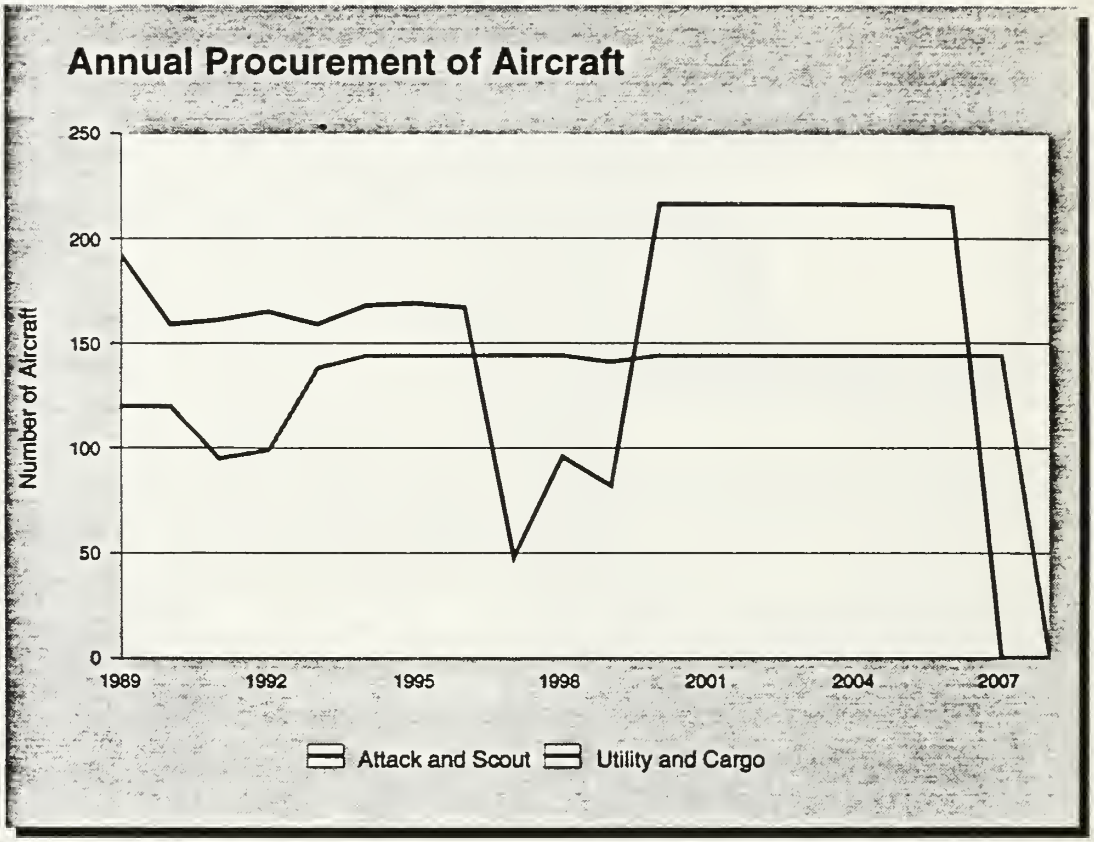


Figure 20. Aircraft Procurement Data for Combined Roles. This graph illustrates the capability to collect data from four roles into two combinations. The analysis principle of extensibility allows for this user controlled addition to the analytical tools and backward compatibility would ensure that this data change would be filtered throughout the graphs and spreadsheets in all previous runs.

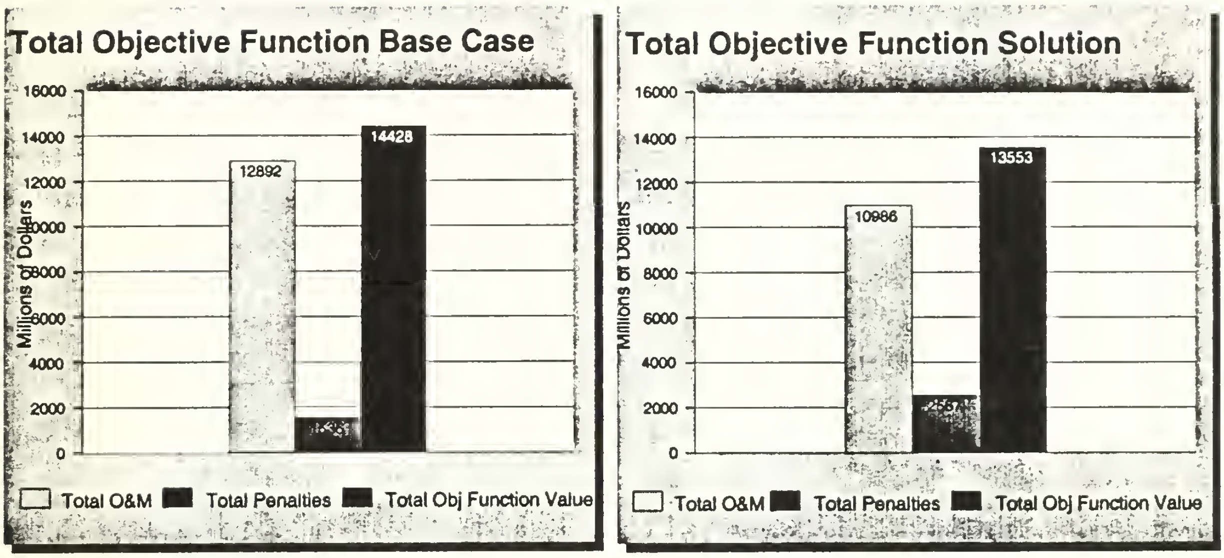
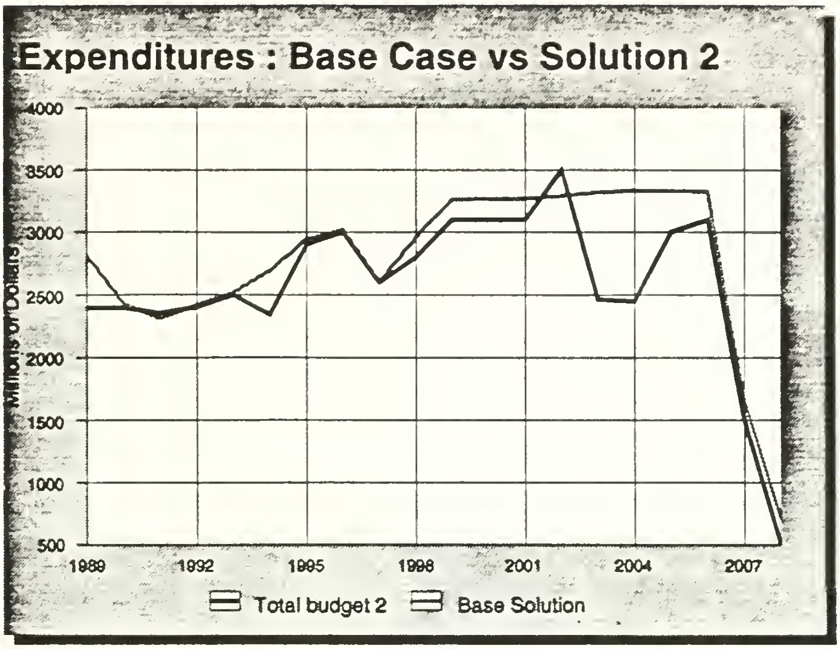


Figure 21a. Two Case Comparison of Objective Function Values. Multiple run analysis is enhanced by the ability to create side-by-side comparisons of subsequent runs to the same information from a base case.

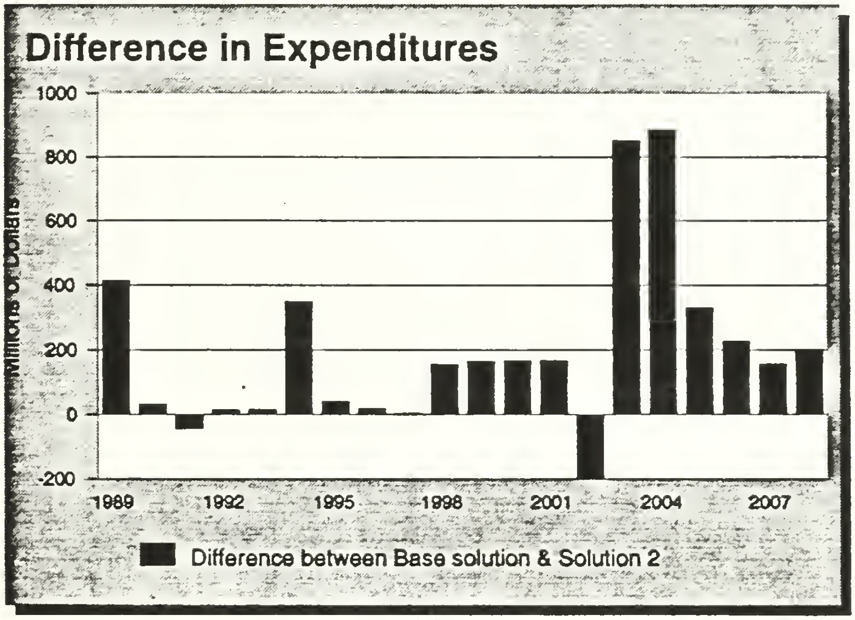
		Purchase Cost
Attack	AH-1S	0
	AH-58D	5.54
	AH-64	13.15
	AH-64B	14.65
	LHX-ATK	7.5
Scout	OH-6	0
	OH-58A	0
	OH-58C	0
	OH-58D	5.23
	LHX-SCT	7.5
Utility	UH-1H	0
	EH-1	0
	UH-60	5.01
	UH-60B	6.51
Cargo	CH-47A	0
	CH-47B	0
	CH-47C	0
	CH-47D	5.09
	CH-54A	0
	CH-54B	0

		Purchase Cost
Attack	AH-1S	0
	AH-58D	5.54
	AH-64	13.15
	AH-64B	14.65
	LHX-ATK	9
Scout	OH-6	0
	OH-58A	0
	OH-58C	0
	OH-58D	5.23
	LHX-SCT	9
Utility	UH-1H	0
	EH-1	0
	UH-60	5.01
	UH-60B	6.51
Cargo	CH-47A	0
	CH-47B	0
	CH-47C	0
	CH-47D	5.09
	CH-54A	0
	CH-54B	0

Figure 21b. Two Case Comparison of Purchase Cost. Multiple run comparisons can be conducted in the spreadsheet environment as well as graphically. This supports the analysis principles of base case and side-by-side comparisons and, in conjunction with figure 21a, that of multiple representations.



22(a)



22(b)

Figure 22. Multiple Run Comparisons. In figure 22a a simple line graph serves to compare total budget expenditures for a base case to an alternate solution. Figure 22b is a difference bar chart supported by a formula that subtracts the expenditures for the alternate solution from those of the base case.

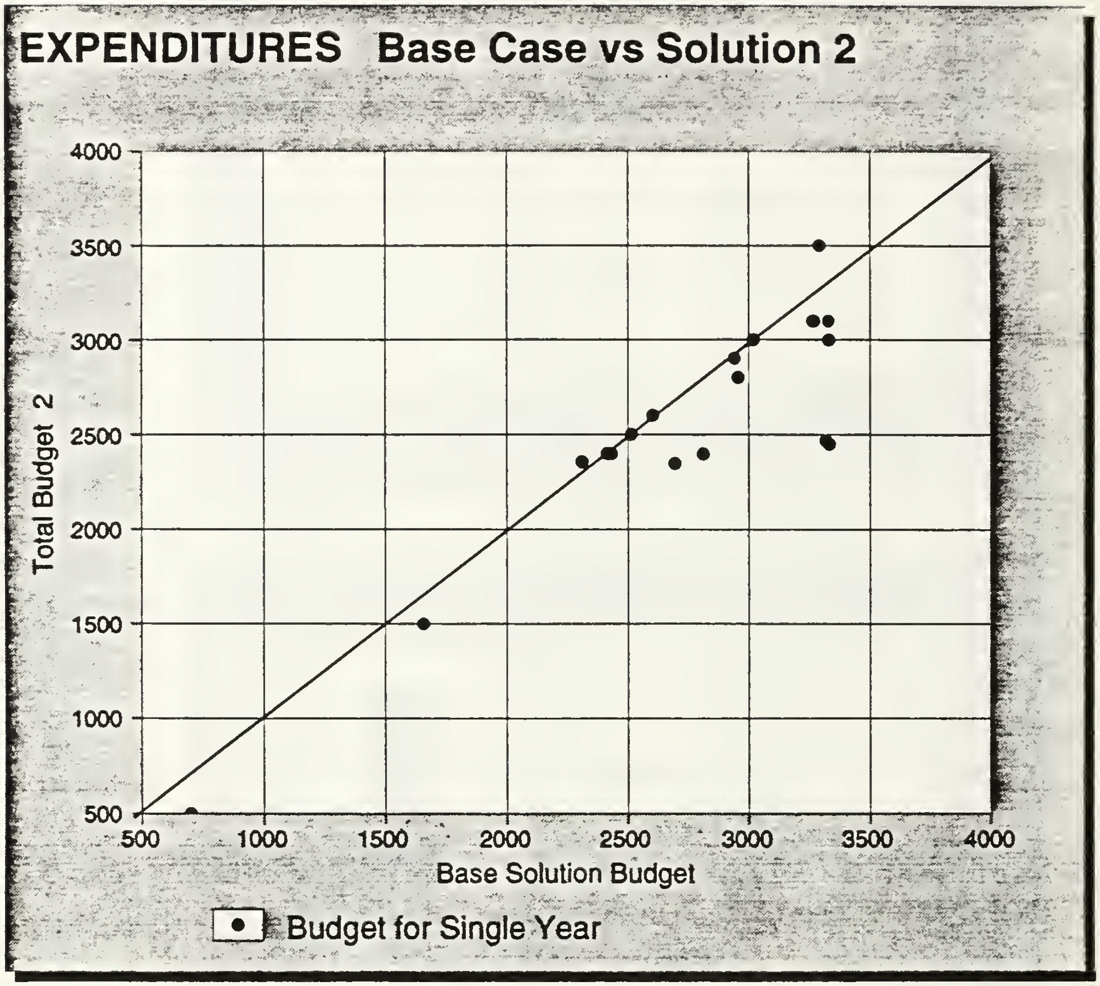


Figure 23. Scatter Plot Comparison. In this comparison of a base case to an alternate solution deviations from the identity (x=y) line are readily apparent. A majority of the points to the right of and below the line indicate larger values for the base solution associated with the horizontal axis.

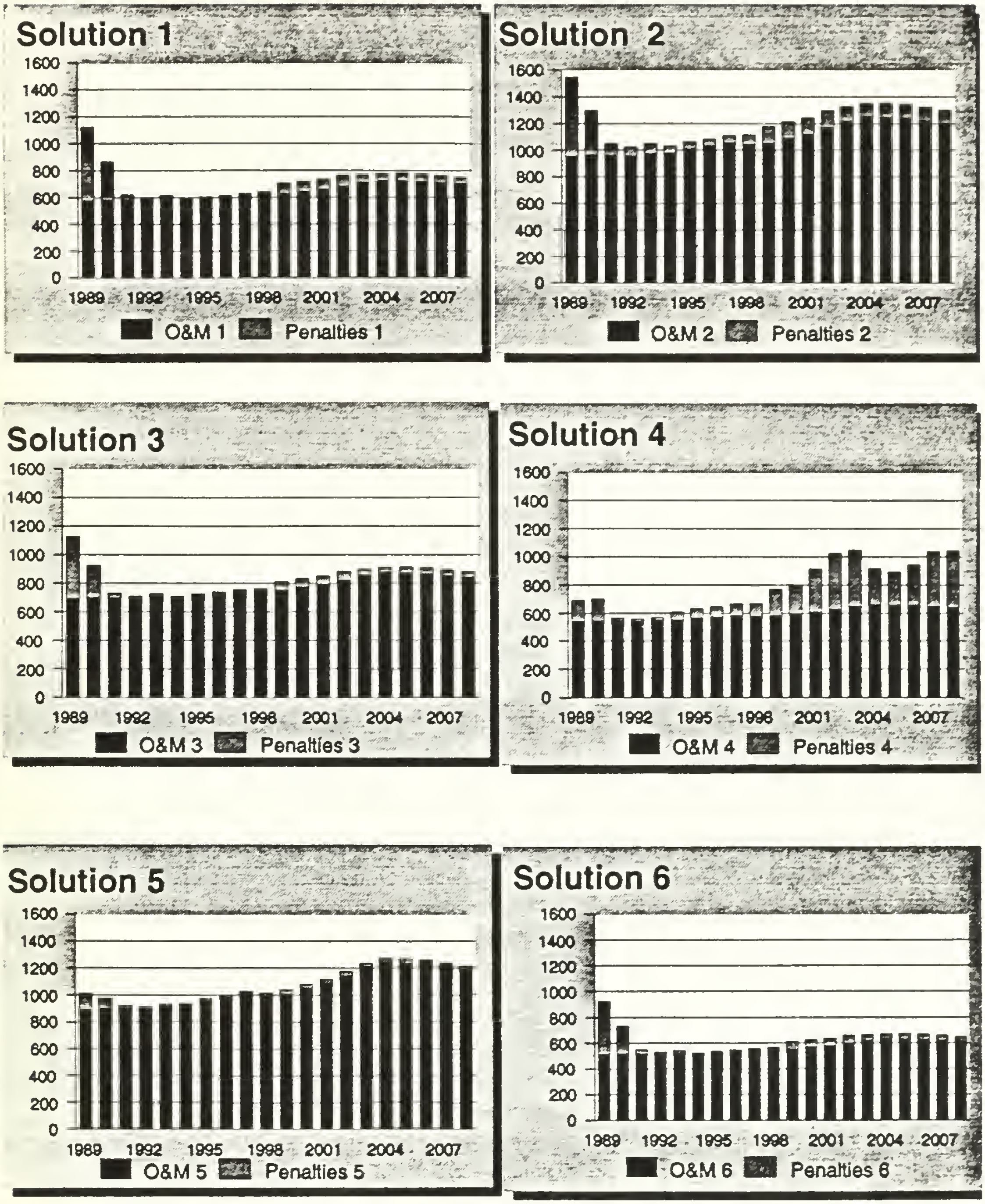


Figure 24. Objective Function Values for Multiple Solutions. The use of multiples facilitates the display and comparison of a variety of solutions with differences and similarities readily apparent.

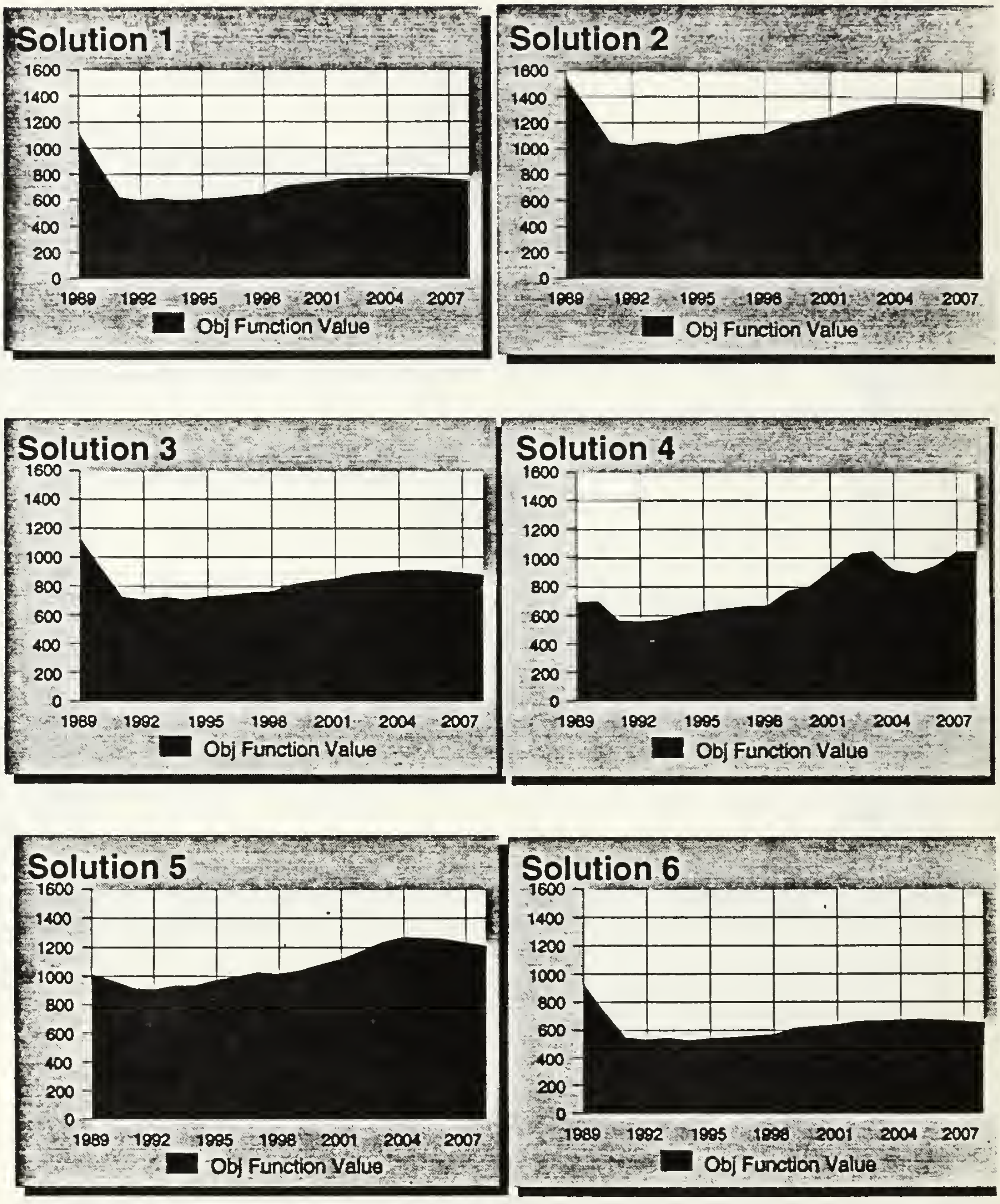


Figure 25. Multiples Displayed with Area Graphs. The use of area graphs presents the objective function values in a more dramatic visualization with the same focus on similarities and differences.

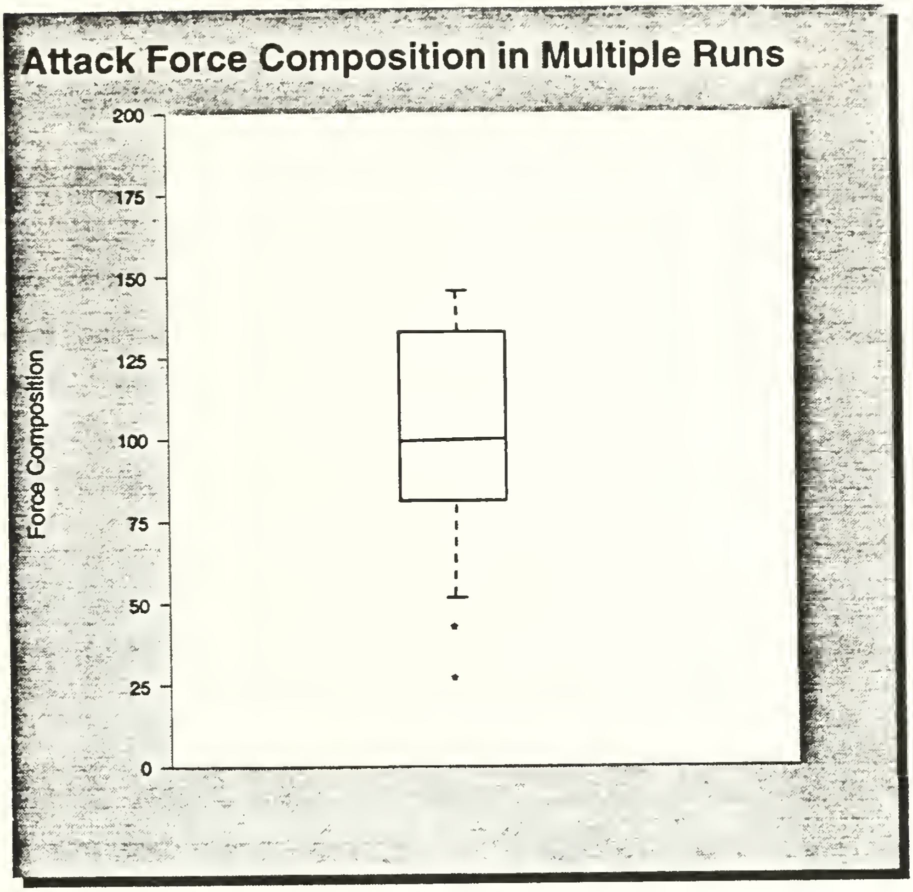


Figure 26. Boxplot. Statistical tools, such as the boxplot, can be used to compare results of several runs of the model. Results can be used to tighten or restrict constraints without effecting the rest of the model. The horizontal line segment inside each box is the 50th percentile and the top and the bottom of the box indicate the 75th and 25th percentiles. The ends of the vertical lines are the adjacent values which represent the largest or smallest observations within 1.5 times the difference between the 75th and 25th percentile on either end of the box. The dots above or below these lines represent outlier values.

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